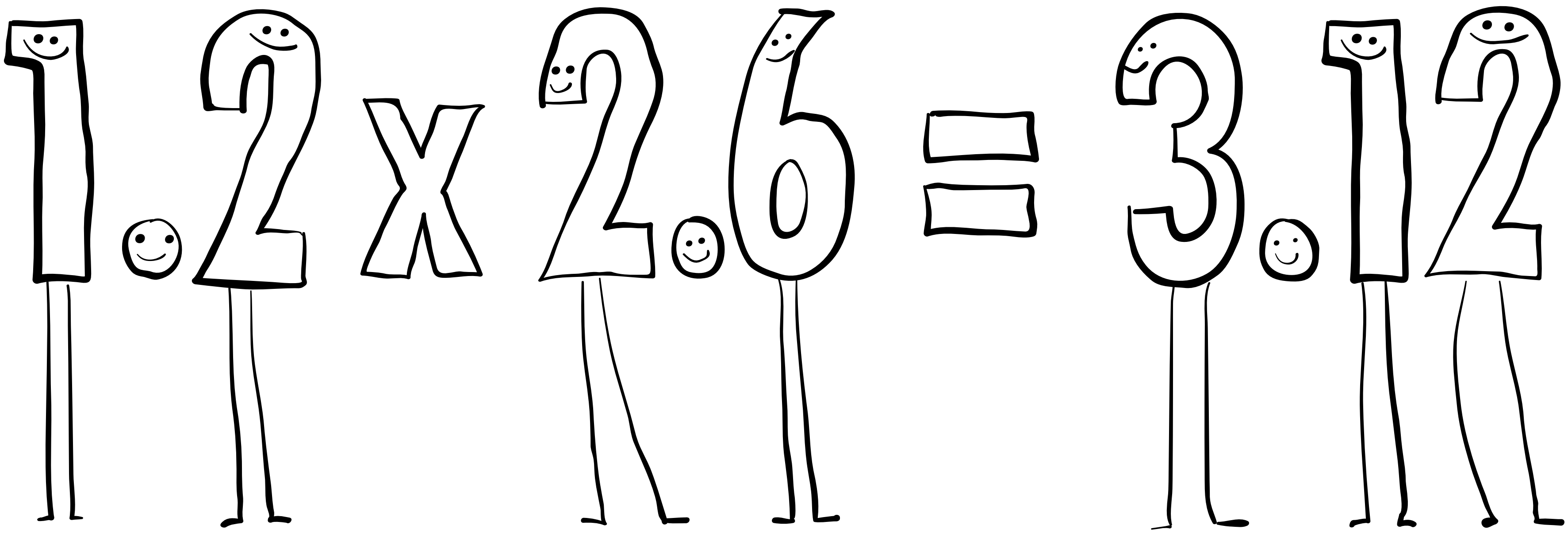


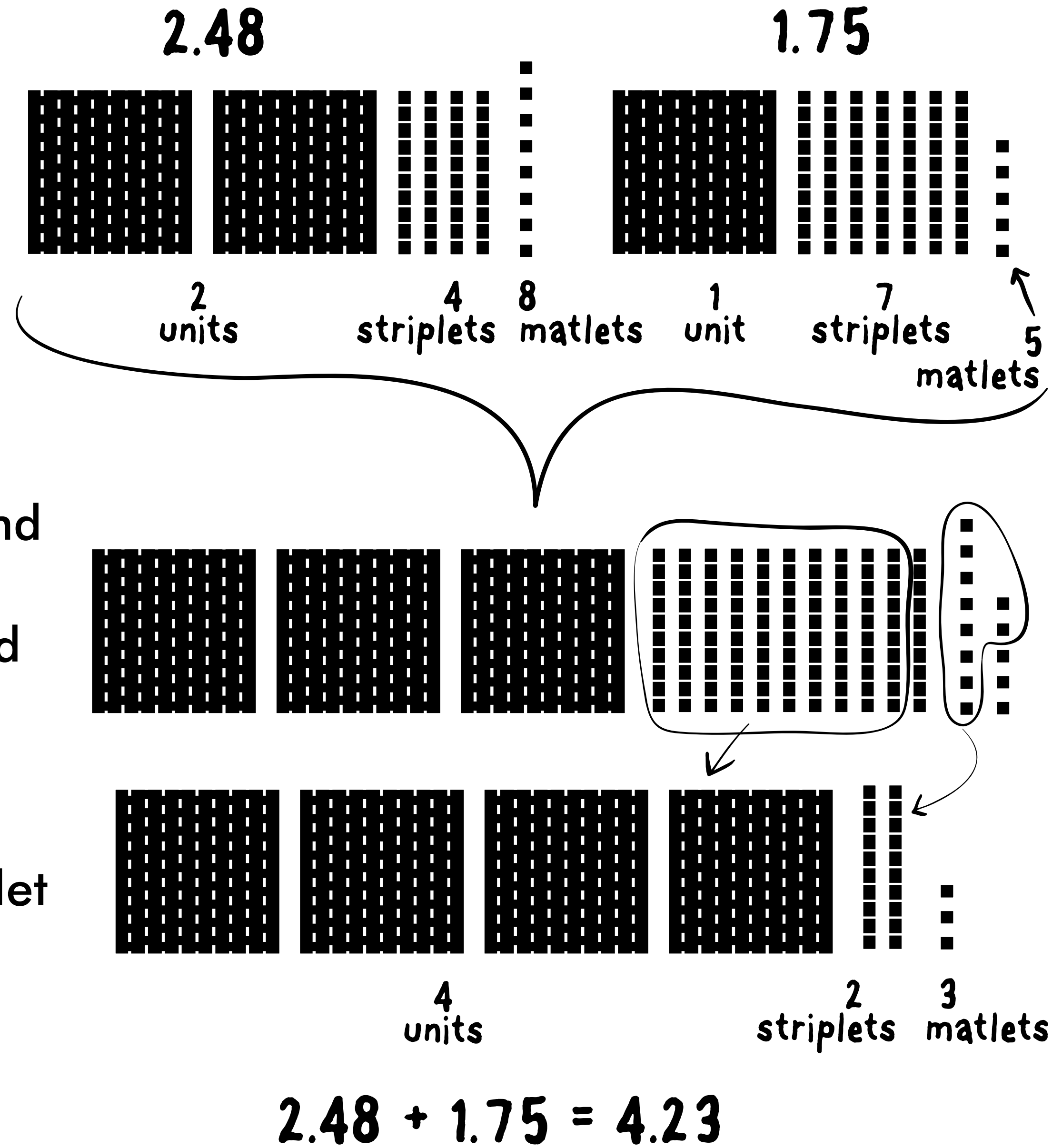
DECIMAL OPERATIONS

The fourteenth scene in a series of articles
on elementary mathematics.

written by Eugene Maier
designed and illustrated by Tyson Smith

Operations with decimals can be modeled with tile pieces, similarly to the manner in which the operations on counting numbers were modeled in Scenes 2,3 ,4 and 5.

To add 2.48 and 1.75 one can combine their representative tile piece collections, convert the combined collection to a minimal collection and record the results. A collection for 2.48 contains 2 units, 4 striplets and 8 matlets and that for 1.75 contains 1 unit, 7 striplets and 5 matlets. The combined collection contains 3 units, 11 striplets and 13 matlets. Trading 10 matlets for 1 striplet and 10 striplets for a unit provides a minimal collection containing 4 units, 2 striplets and 3 matlets. Hence 2.48 + 1.75 = 4.23.



One way to record the type of actions carried out on the previous page is to write the numbers to be added one above another so that the digits representing pieces of the same type are aligned vertically. Then one can combine pieces, from the smallest pieces to the largest, trading for larger pieces when possible. The pieces obtained in a trade are added to the existing pieces while the number of pieces remaining after a trade are recorded in the appropriate column in the sum.

In the example shown here 20 of the 23 matlets are traded for 2 striplets, which are recorded in the striplet column, while the remaining 3 matlets are recorded in the sum. There are now 14 striplets, 10 of which are traded for a unit which is recorded in the units column while the remaining 4 striplets are recorded in the sum. There are now 15 units, 10 of which are traded for a strip which is recorded in the strip column while the remaining five units are recorded in the sum. Finally, the 2 strips are recorded in the sum.

$$
\begin{array}{lcccc}
 & \text{strips} & \text{units} & \text{striplets} & \text{matlets} \\
 & 1^{1} & 2^{1}. & 4^{2} & 8 \\
 & & 7. & 5 & 6 \\
+ & & 5. & 3 & 9 \\
\hline
\text{sum:} & 2 & 5. & 4 & 3
\end{array}
$$

$$12.48 + 7.56 + 5.39 = 25.43$$

To subtract decimals one can either use the take-away model or the difference model. Figure 1 uses a take-away model to find 3.75 – 1.28. Figure 2, on the next page, uses a difference model.

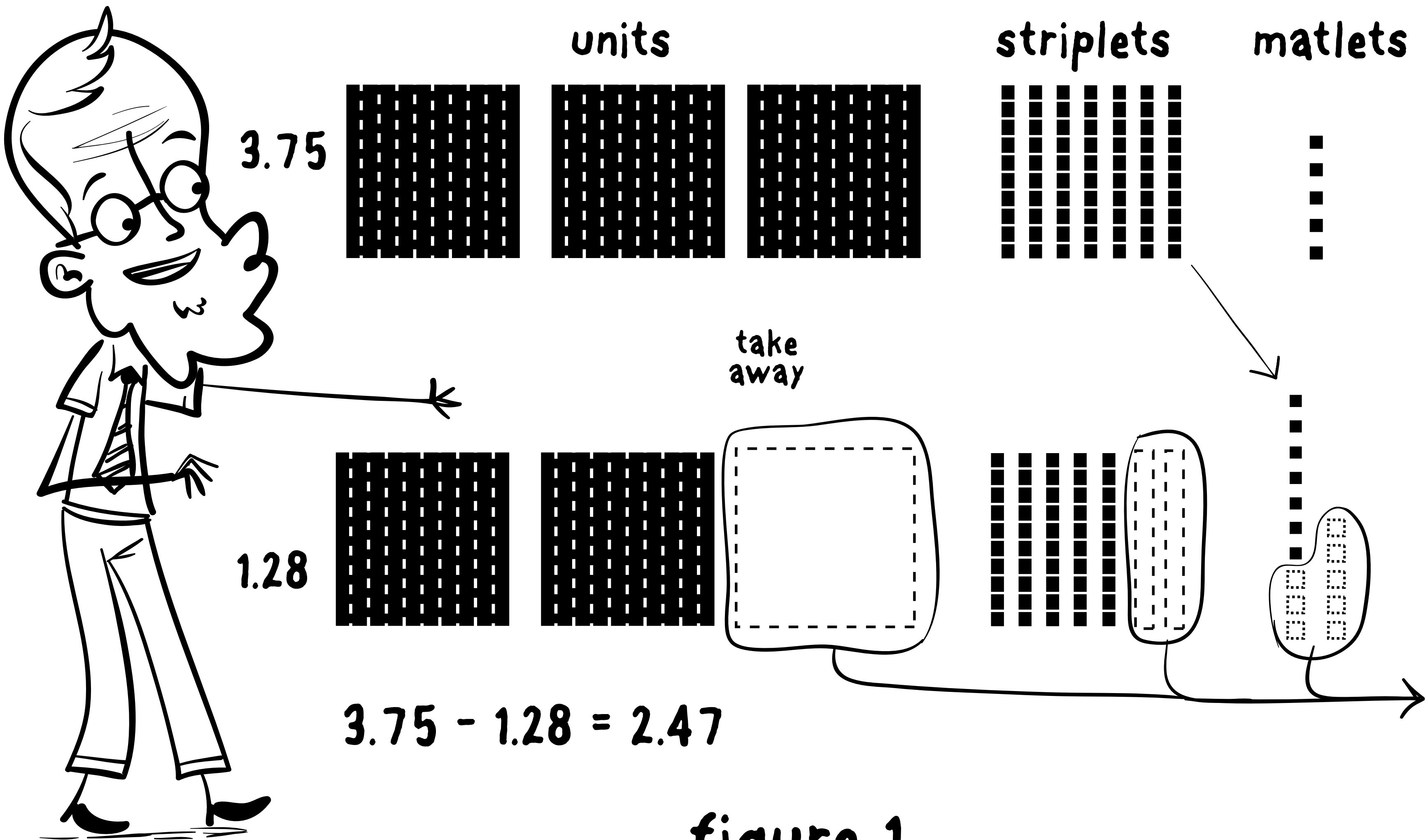
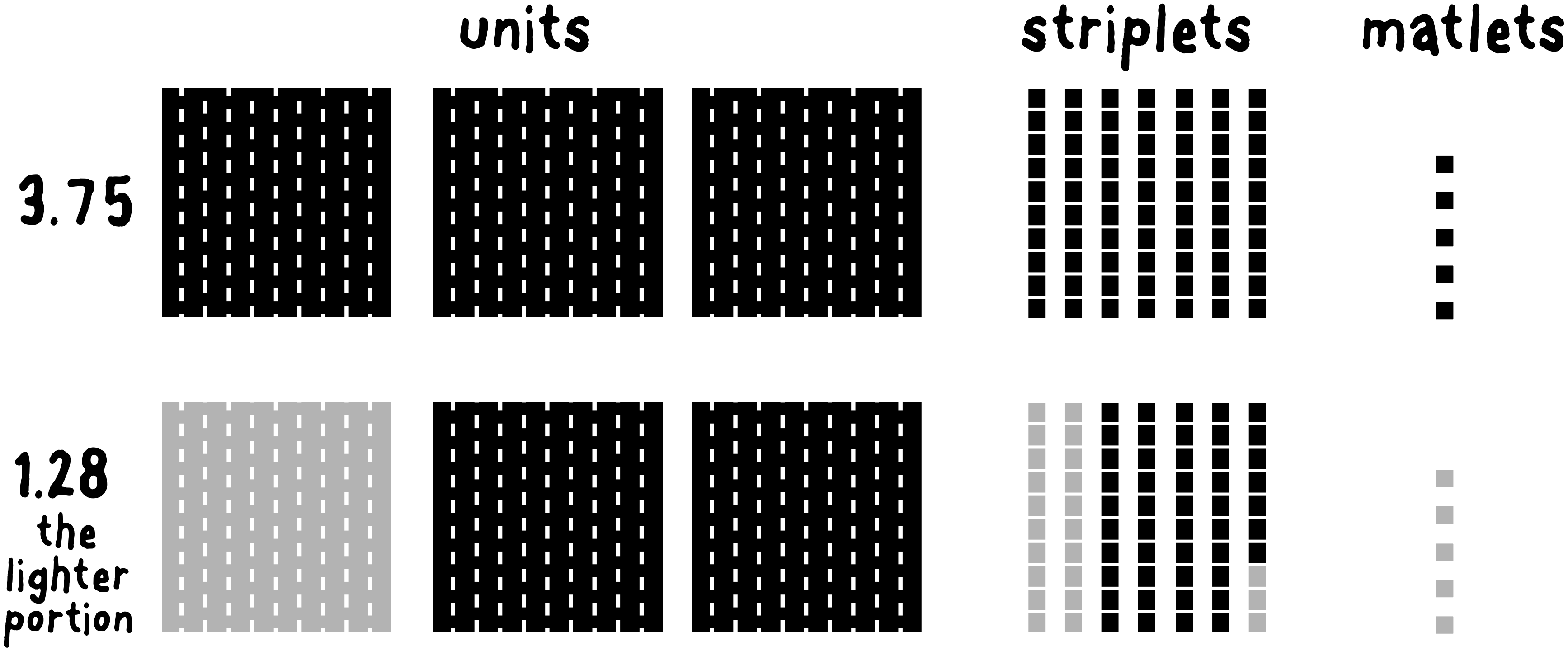


figure 1



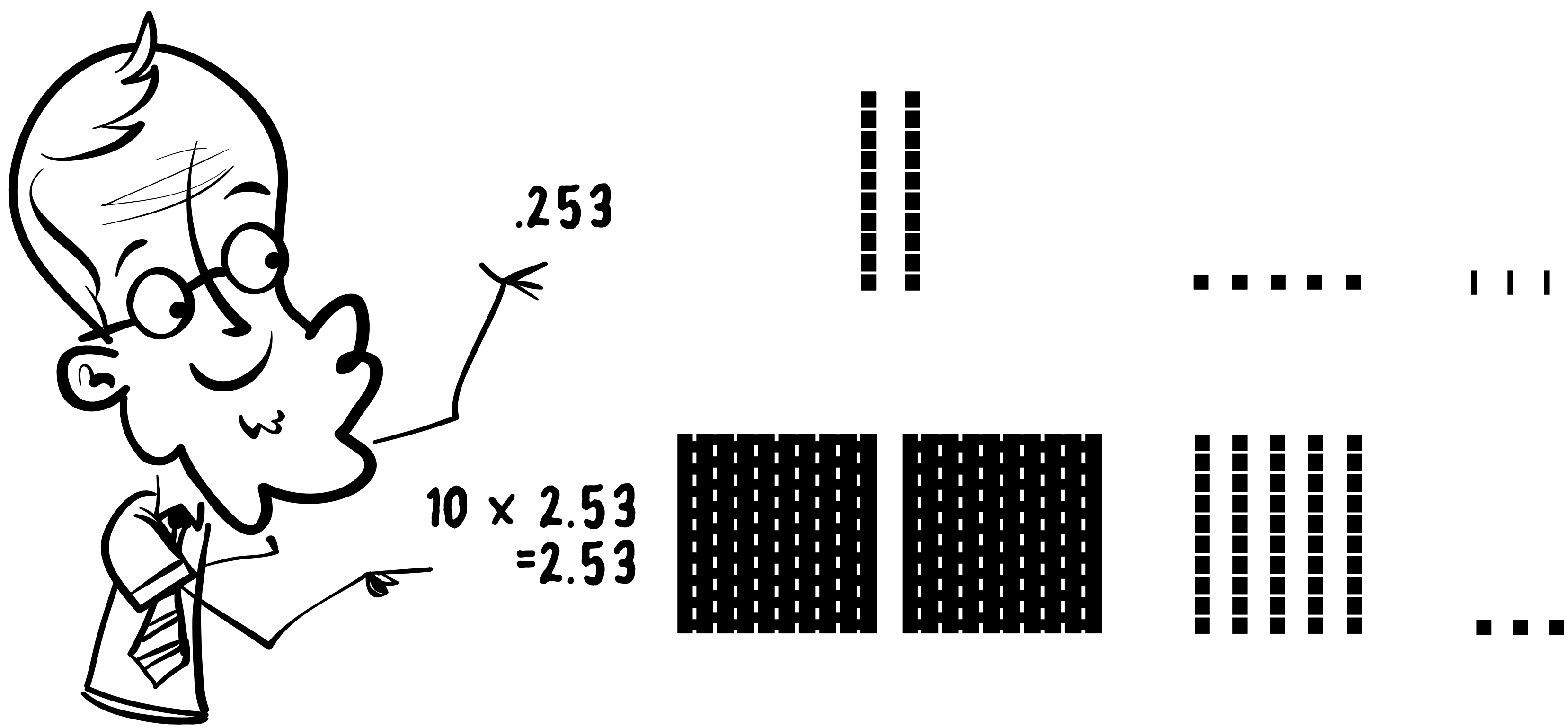
the difference 3.75 - 1.28 is the darker portion

$$3.75 - 1.28 = 2.47$$

figure 2

We begin our discussion of the multiplication and division of decimals with the special case in which a decimal is multiplied or divided by 10.

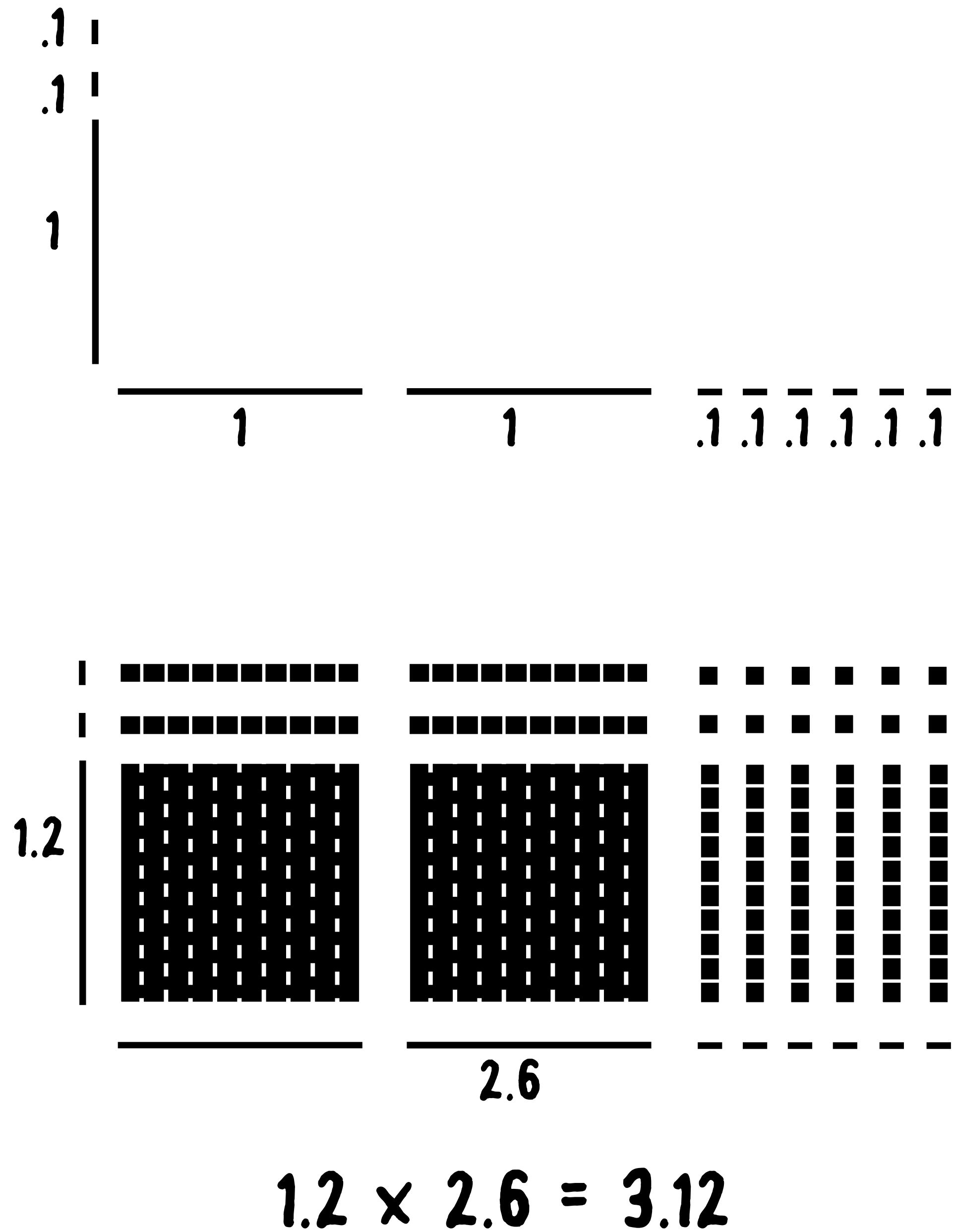
Notice that taking 10 copies of a collection has the effect of replacing every piece in a collection by the next larger piece. As shown below, 10 copies of the collection consisting of 2 striplets, 5 matlets, and 3 strip-matlets is equivalent to the collection consisting of 2 units, 5 striplets and 3 matlets. Thus, 10 x .253 = 2.53.



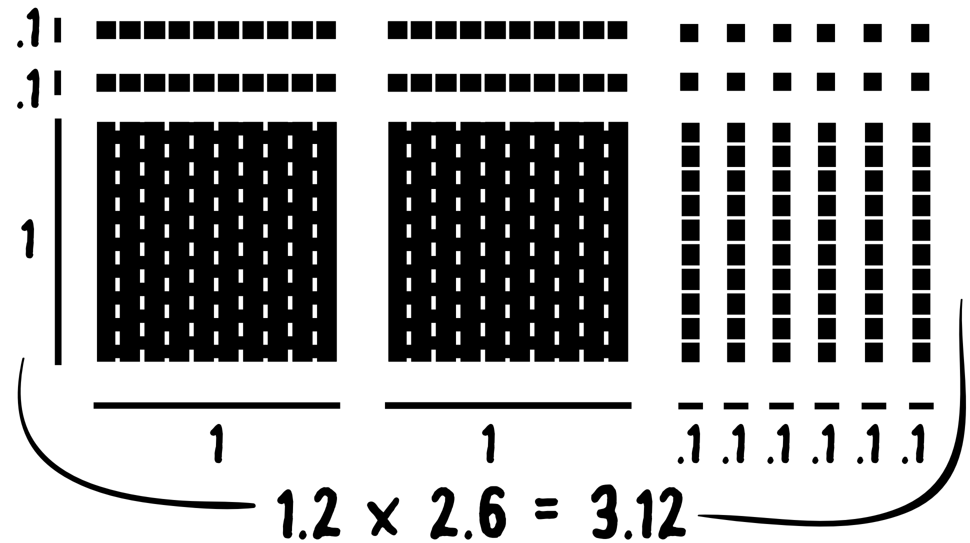
Conversely, if we begin with a collection and divide it into 10 equal parts, one of these parts is identical to the original collection with each piece replaced by the next smaller piece. If the collection for 2.53, shown on the previous page, is divided into 10 parts, one of these parts is a collection for .253. Thus, $2.53 \div 10 = .253$.

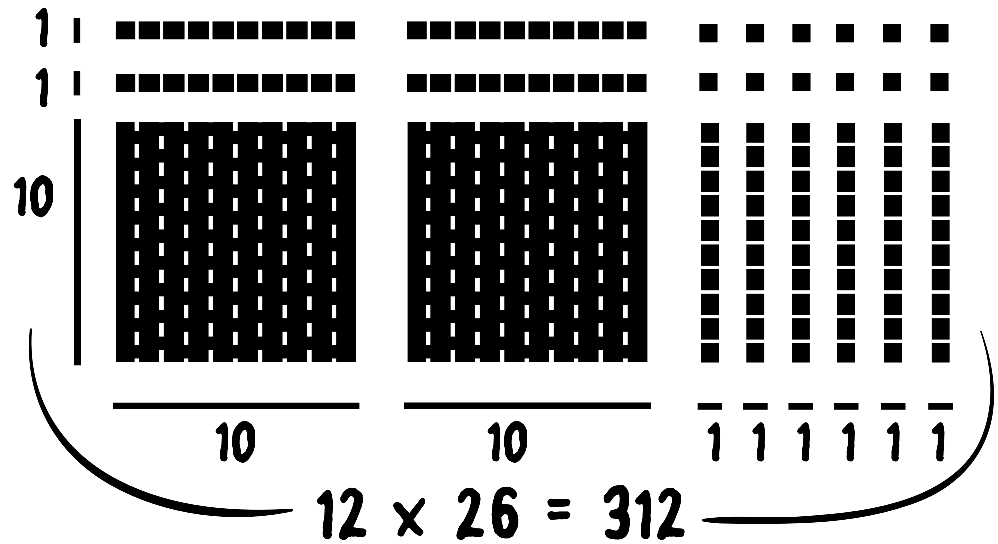
The above observations leads to the oft-stated rule that multiplying a decimal by 10 moves the decimal point one place to the right. Since multiplying by 100 can be accomplished by two successive multiplications by 10, multiplying a decimal by 100 moves the decimal point 2 places to the right; multiplying by 1000 moves the decimal 3 places to the right, and so forth. Similarly, dividing a decimal by 10 moves the decimal point one place to the left, dividing by 100 moves the decimal point 2 places to the left, dividing by 100 moves it 3 points to the left, etc.

In general, as with other kinds of numbers, the multiplications of decimals can be modeled by arrays with edge pieces. Consider, for example, the product 1.2 x 2.6. We begin by laying out edge pieces as shown (the edge pieces of length .1 are obtained by dividing edge pieces of unit length into 10 equal parts). Next, the array is completed using unit pieces, striplets and matlets. The completed array contains 2 units, 10 striplets, which can be traded for a unit and 12 matlets, of which 10 can be traded for 1 striplet. Thus, a minimal collection for the array contains 3 units, 1 striplet and 2 matlets. Hence, 1.2 x 2.6 = 3.12.



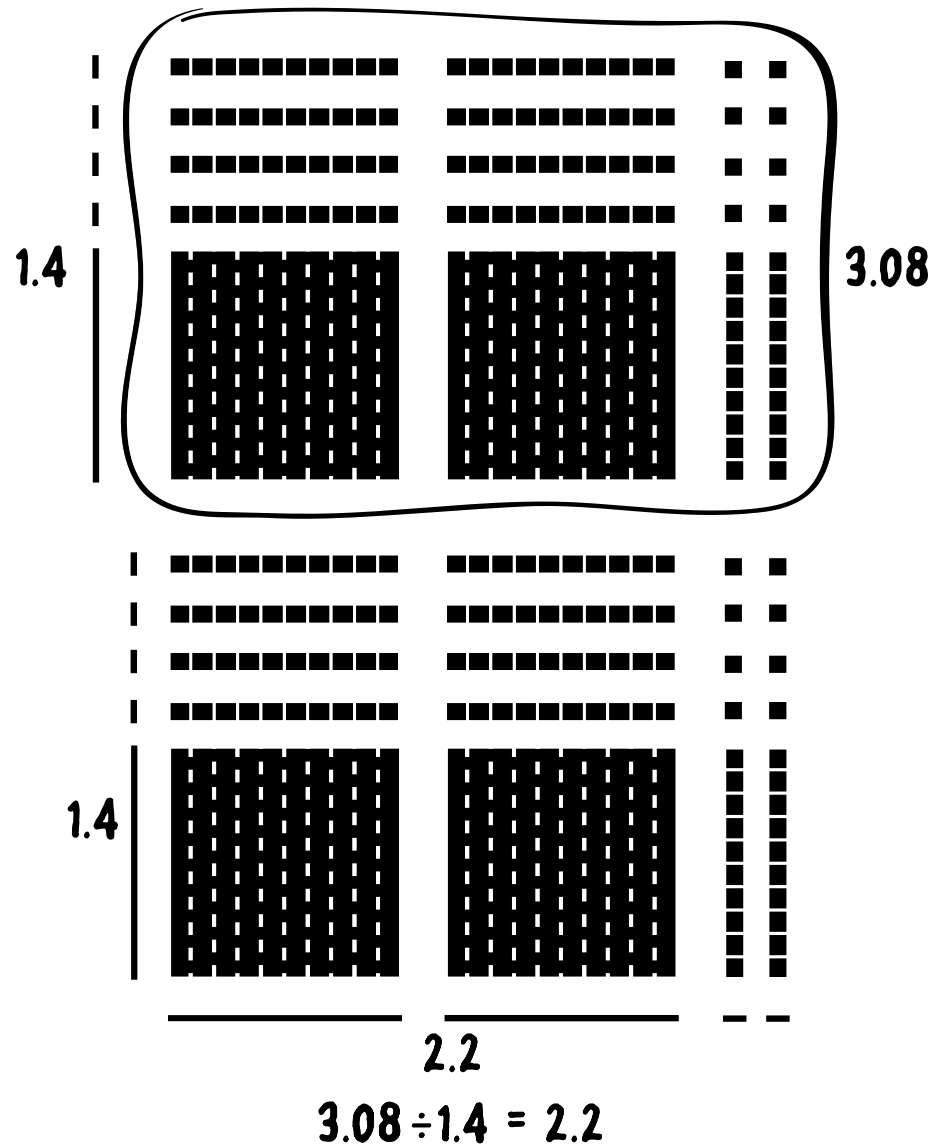
Note that the array for the product 1.2 x 2.6 looks like an array for the product 12 x 26—the arrays contain the same number of pieces, the only difference in the arrays is the value of the pieces. Thus an alternate way of determining the product 1.2 x 2.6 is to compute the product 12 x 26 and then place the decimal point appropriately: since 12 x 26 = 312 and 1.2 x 2.6 is between 1 x 2 and 2 x 3, 1.2 x 2.6 = 3.12





FYI: Multiplying decimals can be accomplished by writing the decimals as fractions, multiplying the fractions as discussed in Scene 11 and then rewriting the result as a decimal, e.g., $1.2 \times 2.6 = \frac{12}{10} \times \frac{26}{10} = \frac{312}{100} = 3.12$.

The division of decimals can also be modeled by arrays with edge pieces. To determine 3.08 ÷ 1.4, we form an array whose value is 3.08 and has one edge whose value is 1.4 and determine the value of the other edge. To do this, we lay out an edge comprised of 1 unit length and 4 tenths of a unit and then distribute a collection consisting of 3 units and 8 hundredths into an array with this edge. To accomplish this, we trade one of the units for 10 tenths. The other edge of the completed array contains 2 edge pieces of length 1 and 2 of length .1. Thus, 3.08 ÷ 1.4 = 2.2.



As is the case with decimal multiplication, we could have determined the quotient and then placed the decimal appropriately: since 308 ÷ 14 = 22 and 3.08 ÷ 1.4 is about 2, 3.08 ÷ 1.4 = 2.2.

END of SCENE 14: DECIMAL OPERATIONS

For comments and questions please email Gene Maier at...
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