visual mathematics

COURSE I STUDENT ACTIVITIES

This packet contains one copy of each Follow-up and of other activities used by individuals or pairs of students. Group activities and sheets are not included.

Visual Mathematics, Course I
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Student Activities

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Student Activities VISUAL MATHEMATICS COURSE I

Follow-up Student Activity 1.1

NAME _____ **DATE** _____

Write a one to two page Mathography that describes your past feelings and experiences in math and that explains your hopes for this math class. Include:

- how you feel about math;
- situations both in and out of school that were "important moments" for you because they affected how you feel about math; and
- what you hope to gain from this class and what you hope to contribute.

My Mathography

Follow-up Student Activity 2.1

NAME _____ **DATE** _____

Problem	Draw a picture of tile and/or linear units to show the meaning of the problem.	Write a word problem whose solution is modeled by your picture.
1 $7 + 9 = 16$		
2 $15 - 6 = 9$		
3 $8 \times 5 = 40$		
4 $24 \div 6 = 4$		

(Continued on back.)

Follow-up Student Activity (cont.)

5 On grid paper, draw a diagram of tile or linear pieces to model the mathematical relationships in each of these situations.

a) Lewis saved $23 last week, which is $8 more than Joanne saved.

b) Adela sold 3 times as many cookies as Josh, who sold 13 boxes.

c) LaTina planted a rectangular garden with area 32 square feet. One side of the garden has length 8 feet.

6 Next to each situation you modeled in Problem 5 write a math question about the situation that could be answered by looking at your model. Then give the answer to your question.

7 On a separate sheet write a letter to a friend who isn't in your math class and tell him or her about the models your class explored for the four basic operations (add, subtract, multiply, and divide). Use clear diagrams and careful explanations to help them understand the meanings of each operation.

Follow-up Student Activity 3.1

NAME _____ **DATE** _____

1 For each of the following equations, draw diagrams of tile that show the meaning of the expression on each side of the equals sign.

a) $4 \times 5 = 5 \times 4$

b) $(2 + 6) + 3 = 2 + (6 + 3)$

c) $4 + 3 = 3 + 4$

d) $3 \times (2 + 3) = (3 \times 2) + (3 \times 3)$

e) $2 \times (5 - 1) = (2 \times 5) - (2 \times 1)$

(Continued on back.)

Follow-up Student Activity (cont.)

2 Jamie wrote each of the following computations to describe his actions with tile. For each computation, draw a diagram to show what you think Jamie's actions were in the order he did them. Next to each diagram, write an explanation of Jamie's actions.

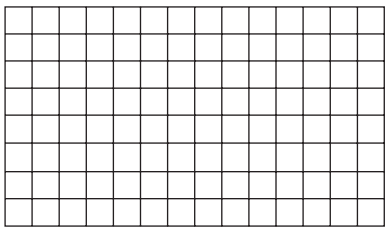
a) $(7 + 9) - (3 + 8)$

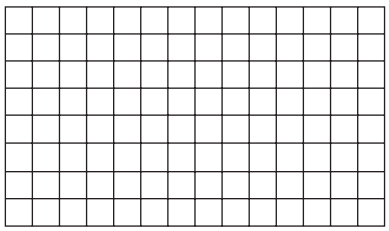
b) $3 + (4 \times 2)$

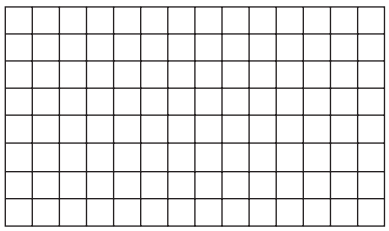
c) $(3 \times (5 - 2)) + 1$

d) $3 \times (4 + 1)$

3 Separate each of these 8×14 rectangles into smaller rectangles to show *3 different* ways to "see" that $8 \times 14 = 112$. Find the area of each 8×14 rectangle by adding the areas of the small rectangles.

a) 

b) 

c) 

Complete these number statements to show how you "saw" and computed the area of each rectangle above.

a) $8 \times 14 =$ _____ b) $8 \times 14 =$ _____ c) $8 \times 14 =$ _____

Follow-up Student Activity 4.2

NAME _____ **DATE** _____

1 Carlos said that he determined the number of tile in the picture at the right by finding (4 + 2) + 5. Maria said she thought about it this way: 4 + (2 + 5). Who was right? Why?

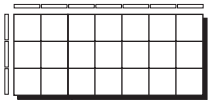
2 Next to each of the following diagrams:

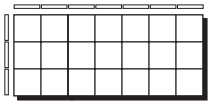
- Write an equation that represents a different method of "seeing" and counting the number of tile in the diagram.
- Subdivide each diagram to illustrate your methods.

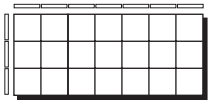
a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

i) 

(Continued on back.)

Follow-up Student Activity (cont.)

3 Use the rules for order of operations that we discovered in class to solve these computations. Next to each computation, write the answer. Then write an explanation of each step you used to get the answer.

a) $10 \times 7 - 12 \times 5 =$

b) $4 + 2 \times 5 - 12 \div 6 =$

c) $8 \div 4 \times (2 + 3) - 4 + 7 =$

d) $12 \times (8 - 6) - 8 \times 0 + 1 =$

e) $13 - 6 \div 2 \div 3 =$

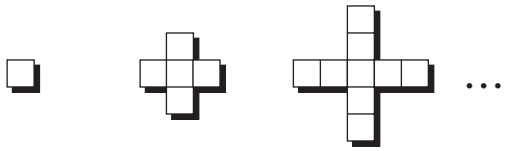
f) $24 + 8 \div 4 - 6 + 14 =$

4 Invent a new set of rules for order of operations. On another sheet of paper, explain what your rules are, and then write the answers to d), e), and f) above using your rules.

Follow-up Student Activity 5.1

NAME _____ **DATE** _____

1 The first 3 figures in a pattern are shown below. Cut out squares and form what you think is the 4th figure. Sketch your 4th figure below.

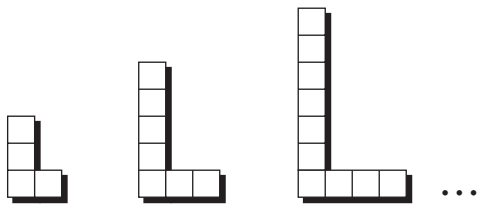


a) Assuming your pattern continues, explain how you think these 3 figures give you clues to what the 50th figure looks like.

b) Tell how (other than building the figure and counting tile) to find the total number of tile in the 50th figure.

c) Describe another method (other than building and counting) of finding the number of tile in the 50th figure of the pattern above.

2 The first 3 figures in another pattern are shown below. Form what you think is the 4th figure. Draw your 4th figure below.



Assuming your pattern continues, explain *two* different methods (other than building the figure and counting) of telling what the 35th figure looks like and how many tile it contains.

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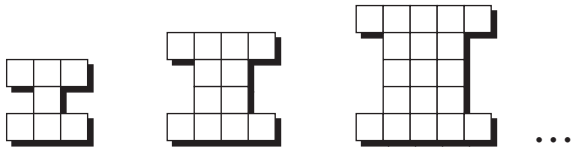
Follow-up Student Activity (cont.)

3 The first 3 figures in another pattern are shown below:



a) Tell how many tile you think are in the 70th figure and explain how you decided this number.

b) Tell another method (other than building and counting) of finding the number of tile in the 70th figure.

c) Suppose a certain figure in the above pattern has exactly 444 square tile in it. Which figure is it? Explain how you decided this.

4 Create the first 4 figures in an interesting pattern of tile figures. Sketch your 4 figures below.

5 Describe your pattern in Problem 4 and tell what the 20th figure in your pattern looks like.

6 Cut out squares from the attached grid paper and build the first 3 figures in the pattern in Problem 1. Get an adult to share with you how they "see" the 20th figure in the pattern. Repeat this process for Problems 2 and 3. If needed, help the adult by sharing ways you "see" each 20th figure. On another sheet, describe what happened.

Follow-up Student Activity 6.1

NAME _____ **DATE** _____

1 A student said that "any collection of tile that can be arranged to form a rectangle with no gaps or overlaps must contain an even number of tile." Is the student correct? Explain and draw diagrams to show why you think this way.

2 Draw diagrams of these numbers so that it is possible to "see" without counting whether each number is odd or even.

a) 27 b) 32 c) 19

3 For each of the following, draw a picture to show why you think the answer is *always* odd, *always* even, or *sometimes* odd and *sometimes* even.

a) the sum of 2 odd numbers

b) the sum of an even number and an odd number

c) the difference between an odd an even number

d) the difference between 2 odd numbers

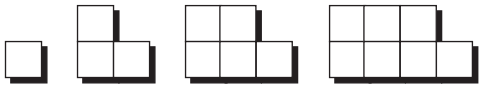
4 Is the sum of two consecutive counting numbers *always* odd, *always* even, or *sometimes* odd and *sometimes* even? (Note: consecutive means one comes right after the other.)

Draw a diagram to show why you feel your answer is correct.

(Continued on back.)

Follow-up Student Activity (cont.)

5 According to the model explored in class, the first 4 odd numbers look like this:



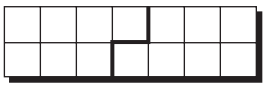
Using this pattern, imagine the odd number that contains 179 tile. Write a description of what it looks like and explain how to tell which odd number it is (the 1st, 2nd, 3rd, etc.).

6 Sketch a diagram of the first 4 even numbers, based on the model explored in class.

7 Imagine the 50th even number in your pattern for Problem 6. Write a description of it and tell how many tile it contains.

8 Imagine the even number with 208 tile. Describe what it looks like and explain how to tell which even number it is (the 1st, 10th, 20th, etc.)

9 This diagram shows that 14 can be divided into 2 equal odd numbers.



What are all the other numbers that can be divided into 2 equal odd numbers? Explain why this is so.

Focus Student Activity 7.1

NAME _____ **DATE** _____

No.	Number of Rectangles	Dimensions of Rectangles	Factors of the Number	
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Follow-up Student Activity 7.2

NAME _____ **DATE** _____

1 Sketch all possible rectangles that can be formed with tile for each of the following numbers. Label the dimensions of each rectangle (it isn't necessary to show every tile in your sketches).

a) 40

c) 48

b) 32

d) 23

2 List all the factors for each number in Problem 1.

a) 40

c) 48

b) 32

d) 23

3 List any numbers in Problem 1 that are prime. Make another list of any that are composite.

Prime:

Composite:

How did you decide whether a number is prime or composite?

(Continued on back.)

Follow-up Student Activity (cont.)

4 Use clues a)-h) below to solve this puzzle. Keep notes of your methods and conclusions as you work (see Problem 6).

a) I am a rectangle.
b) My dimensions are consecutive counting numbers.
c) My perimeter (the total number of linear units around me) has 4 factors.
d) The sum of my length and width is a prime number.
e) One of my dimensions is an odd number and one is even.
f) My area is not a square number.
g) My perimeter is 3 less than a prime number and 1 more than another prime number.
h) My area is less than the 10th square number and greater than the 7th prime number.

Draw a picture of me and label my dimensions, area, and perimeter.

5 Are there any clues that aren't needed to solve the puzzle in Problem 4? List the fewest clues needed in order to be sure you have the correct rectangle.

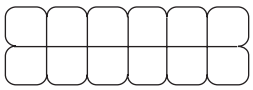
6 On a separate sheet, describe the methods and reasoning that you used to solve problem 4.

7 On another sheet, write a letter to a friend explaining the meaning of the following: factor, prime number, composite number, and square number. Include pictures that will help your friend "see" and understand your explanations.

Follow-up Student Activity 8.1

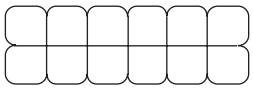
NAME _____ **DATE** _____

1 Subdivide this carton to show sixths and fill 4/6 of the carton.

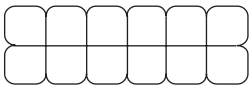


2 On these cartons, show 4/6 of a dozen using twelfths and thirds.

a) use twelfths

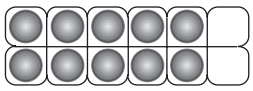


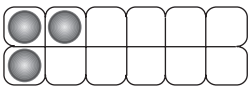
b) use thirds

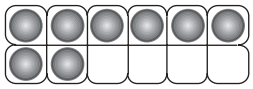


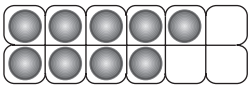
3 List the equivalent fractions shown by your diagrams in Problems 1 and 2.

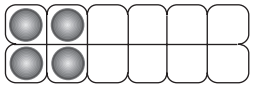
4 For each of Problems a)-i) below, write at least two different fraction names for the part of a dozen that is shown in the diagrams.

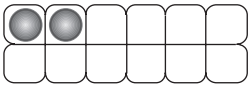
a) _____ 

b) _____ 

c) _____ 

d) _____ 

e) _____ 

f) _____ 

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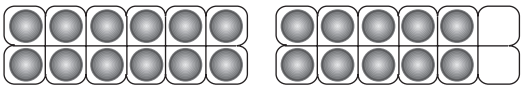
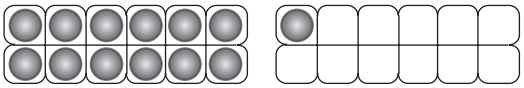
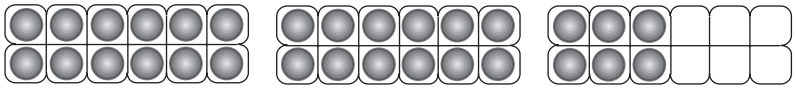
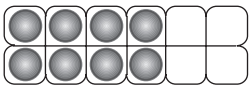
Follow-up Student Activity (cont.)

g) _____ 

h) _____ 

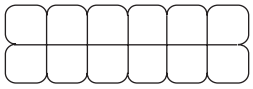
i) _____ 

5 Mackenzie gave $2\frac{2/3}{4}$ (two and two-thirds fourths) as one fraction name for the part of a dozen shown here:



Explain how you think she decided this.

6 Mackenzie named another fraction as $4\frac{1/2}{6}$. Fill in this egg carton to show what part of a dozen you think she was naming:



7 On a sheet of grid paper sketch the following:

a) Two differently-sized cartons which each can be subdivided to show tenths. Fill and label $\frac{4}{10}$ of each carton.

b) A carton which you can subdivide to show both eighths and sixths. Fill and label $\frac{3}{8}$ of the carton and $\frac{1}{6}$ of the carton.

c) Two different ways of viewing the *meaning* of $\frac{3}{5}$. Next to your sketches, write an explanation of how each diagram shows a different meaning of $\frac{3}{5}$.

Follow-up Student Activity 9.1

NAME _____ **DATE** _____

1 On grid paper make "egg carton diagrams" to model each of these situations (use whatever-sized cartons are needed). Then write as many observations as you can about mathematical relationships you can "see" in each of your diagrams.

a) It takes $\frac{1}{6}$ of a dozen eggs to make a gallon of ice cream. Gene has $\frac{3}{4}$ of a dozen eggs.

b) A store manager found a damaged shipment of eggs. The eggs were packed in cartons that each hold 1 dozen eggs. Five cartons each had $\frac{1}{4}$ of their eggs broken. Four cartons each had $\frac{1}{3}$ of their eggs broken. One carton had $\frac{1}{6}$ of its eggs broken. Three cartons had no eggs broken.

c) Eldon and Liz bought identical boxes of candy. Eldon has $\frac{7}{8}$ of a box left and Liz has $\frac{1}{3}$ of a box left.

d) Mark needs $\frac{3}{5}$ box of apples for 1 batch of his special applesauce recipe. He has $2\frac{5}{9}$ boxes of apples.

e) Katrina brought 6 cartons of donuts to share equally among the 5 groups of students.

f) $\frac{2}{3}$ of the earth's surface is covered by oceans and $\frac{1}{10}$ is covered by glaciers.

g) Ted had no money left over after he spent $\frac{1}{2}$ of his year's income on food and rent, $\frac{1}{3}$ of his income on clothing, $\frac{1}{12}$ on entertainment, and saved $1,400.

h) $\frac{2}{3}$ of Ms. Quan's 5th grade class were boys. She sent 4 boys to another class and replaced them with 4 girls. Now $\frac{1}{2}$ of Ms. Quan's class are boys.

i) The 7 girls on the team each received $\frac{4}{7}$ package of socks.

(Continued on back.)

Follow-up Student Activity (cont.)

2 For each of the following, fill in the blanks with fractions or mixed numbers to make a challenging (to you!) fraction computation that you think you could solve by drawing egg carton models. Then show how to solve each problem using an egg carton model (on grid paper).

a) _____ + _____ = _____

b) _____ − _____ = _____

c) (challenge) _____ × _____ = _____

d) (challenge) _____ ÷ _____ = _____

3 Write a word problem that would require each computation you wrote in Problem 2.

4 A question I still have about fractions is…

Follow-up Student Activity 10.1

NAME _____ **DATE** _____

1 The pairs of numbers below represent the heights of stacks of cubes to be averaged (leveled-off). On the grid sketch the front views of columns of cubes with these heights before and after they are leveled-off. Label the heights of all columns.

a) 14 and 8

b) 7 and 12

2 Draw sketches that show how to *use a model or diagram of cubes* to solve each of these puzzle problems. Then write an explanation of your methods and reasoning. (Remember a sketch doesn't need to show each cube.)

a) Andrew scored 21 points during his last basketball game. What must he score during his next game in order to have a 25 point average for the 2 games?

(Continued on back.)

Follow-up Student Activity (cont.)

b) During the first week of a fund raising project Maria sold 18 candy bars. After the second week her average sales for the 2 weeks was 15 candy bars per week. How many candy bars did she sell during the second week?

c) Tyson bowled 2 games last night. The difference between his 2 scores was 18 points. His average score for the 2 games was 167 points. What did he score on each of the 2 games?

3 Write two interesting word problems that involve averaging.

4 Describe one or two ideas related to averaging that you learned or understand better after our class explorations of averaging.

5 What question(s) do you have about averaging?

Follow-up Student Activity 11.1

NAME _____ **DATE** _____

1 For each of the sets of numbers listed below, draw the front views of columns of cubes whose heights are the same as the numbers. Show how to level-off each set of stacks to find the average of that set of numbers. Label the averages on the grid.

a) 9, 12, 4, and 11

b) 5, 15, 8, 9, and 13

2 Make a *sketch* to show the leveling-off of stacks of cubes of heights 105, 92, 102, and 97.

What is the average of 105, 92, 102, and 97?

(Continued on back.)

Follow-up Student Activity (cont.)

3 On a separate sheet make sketches that show how to use a model to solve each of the following puzzle problems.

a) In 4 days of baby-sitting Dan earned $6, $7, $12, and $9. What was the average amount he earned each day?

b) The average of 6 numbers is 9. What is the sum of the numbers?

c) In 3 games Rachelle made 17, 23, and 15 points. What was the average number of points she made per game?

d) Ramon's average bowling score after 3 games was 152. What must he score on the last game to raise his average to 160?

e) After 5 assignments, Marcia's average was 88 points. Her average for the next 2 assignments was 95. What was her overall average for the 7 assignments?

f) Suppose 3 bonus points were added to each of Marcia's assignments in problem e) above. How would that affect her average?

g) Jeremey scored an average of 18 points per game during the first 5 games of the season. During the 6th game he was injured and scored no points. What was his 6-game average?

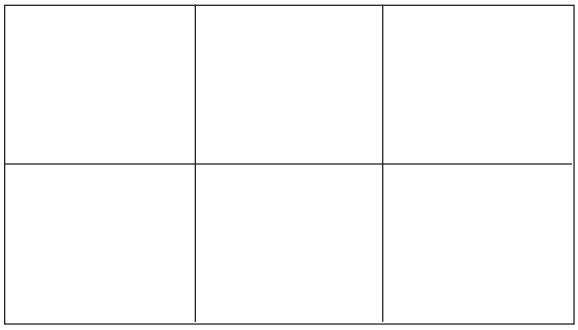
4 The average of 5 numbers is 37. The difference between the largest and smallest number is 12. On another sheet investigate and report all that you can about the 5 numbers.

Focus Master A

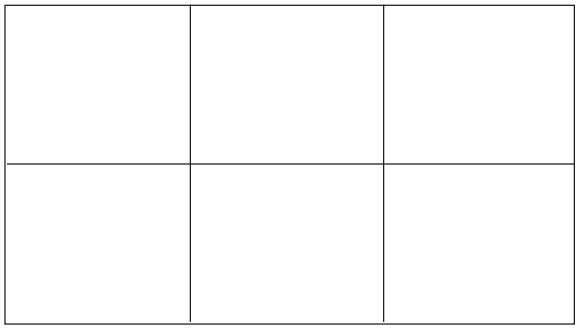
a)

Draw #	Red	Blue	Green
1			
2			
3			
4			
5			
6			
b) Experimental Probability			
Theoretical Probability			

c)



Area Model of Experiment



Area Model of Actual (Theoretical) Contents of Sack

Follow-up Student Activity 12.1

NAME _____ **DATE** _____

1 Imagine that the tile shown in this rectangle are placed in a sack and that 1 tile is randomly selected.

Red	Blue	Red	Green
Green	Red	Blue	Red
Blue	Green	Red	Blue

a) Which color is most likely to be selected from the sack? How did you decide this?

b) What is the *theoretical probability* that the color of the tile selected is: red?_____ green?_____ blue?_____
red or blue?_____ yellow?_____ not blue?_____

2 Cut out the rectangle in Problem 1 and cut apart the squares. Place all 12 squares in a sack or other container and carry out the following experiment.

• Without looking in the sack, select 1 square and record its color here:

Color	Tally Marks
Red	
Blue	
Green	
Yellow	

• Replace the square in the sack and shake the sack to mix the squares.

• Repeat this process 60 times.

(Continued on back.)

Follow-up Student Activity (cont.)

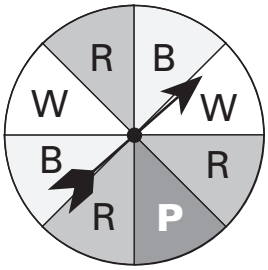
3 Based on the data from your experiment in Problem 2, what is the *experimental* probability that a tile drawn is:
red?_____ green?_____ blue?_____
red or blue?_____ yellow?_____ not blue?_____

4 Using the tile in Problem 2 make up a 2-person game that you think is not fair. Describe the rules of your game on another sheet, and explain why it is not fair and which player has the advantage.

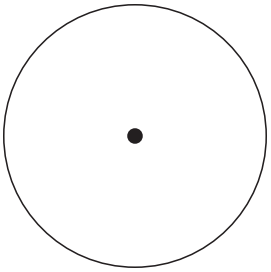
5 A total of 28 red, yellow, and blue tile are placed in a sack. One tile is randomly selected from the sack. The probability of selecting a red tile is ¼. The probability of selecting a yellow tile is twice the probability of selecting a blue tile. Sketch a rectangle at the right showing this collection of tile (mark the color of each tile).

What is the theoretical probability of selecting from your rectangle a tile that is: yellow?_____ blue?_____ not blue?_____ red or blue?_____

6 The spinner at the right has 8 equal parts. Assume the spinner is to be spun 100 times. Predict the approximate number of times you think the spinner will be likely to point to B _____, to P _____, to R *or* B _____, to any letter other than W _____. Tell how you decided each number.



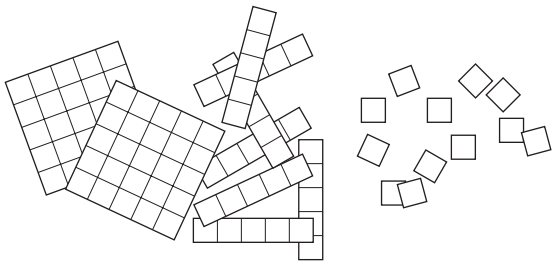
7 Complete the spinner shown at the right so it has 4 parts. Color each part either blue, red, or green so that the probability of spinning a blue is ¼ and the probability of spinning a red is twice the probability of spinning a green.



Follow-up Student Activity 13.1

NAME _____ **DATE** _____

1 Use your base five area pieces to form a minimal collection that contains the same total number of units as the collection below. Sketch the minimal collection you formed.



2 For each number of total units listed in the chart below:

- use your base five pieces to form the minimal collection with the same total number of units;
- record on the chart the number of each type of piece in your minimal collection;
- write a numerical statement that shows the base five notation for the collection.

Total Units	Minimal Collection				Numerical Statement
	Strip-Mats	Mats	Strips	Units	
Example) 115	0	4	3	0	$115 = 430_{five}$
a) 91					
b) 157					
c) 126					
d) 205					
e) 623					

(Continued on back.)

Follow-up Student Activity (cont.)

3 How many total units are in the collection represented by 13214_{five}? Explain your methods for deciding.

4 What do you think a unit, strip, mat, and strip-mat in base four would look like? Sketch pictures of your ideas below.

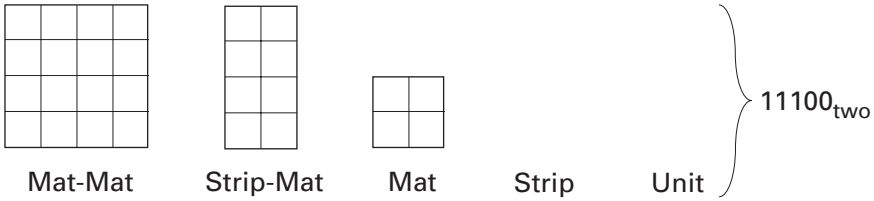
5 Draw a picture of what you think would be the base four minimal collection for 137 total units.

6 What digits do you think are used in base four? Explain your reasoning.

Follow-up Student Activity 14.1

NAME _____ **DATE** _____

1 The base two minimal collection for 28 total units is shown below. It contains 1 mat-mat, 1 strip-mat, 1 mat, 0 strips, and 0 units, and its base two representation is 11100_{two}.



On the grid below sketch the minimal collection of base pieces for 28 total units in each of the following bases. Circle each collection and write its representation in base notation.

a) base five b) base three c) base seven d) base ten

(Continued on back.)

Follow-up Student Activity (cont.)

2 Imagine or draw on the attached grid paper a collection of 3 mats, 5 strips, 6 units, and 2 strip-lets in each of the bases listed below. Then complete the chart below. Remember that for a "sketch" you don't have to show all the grid lines on the pieces.

Sketch of Pieces	Total Units	Base Notation
a) base seven:		
b) base eight:		
c) base twelve:		
d) base ten:		

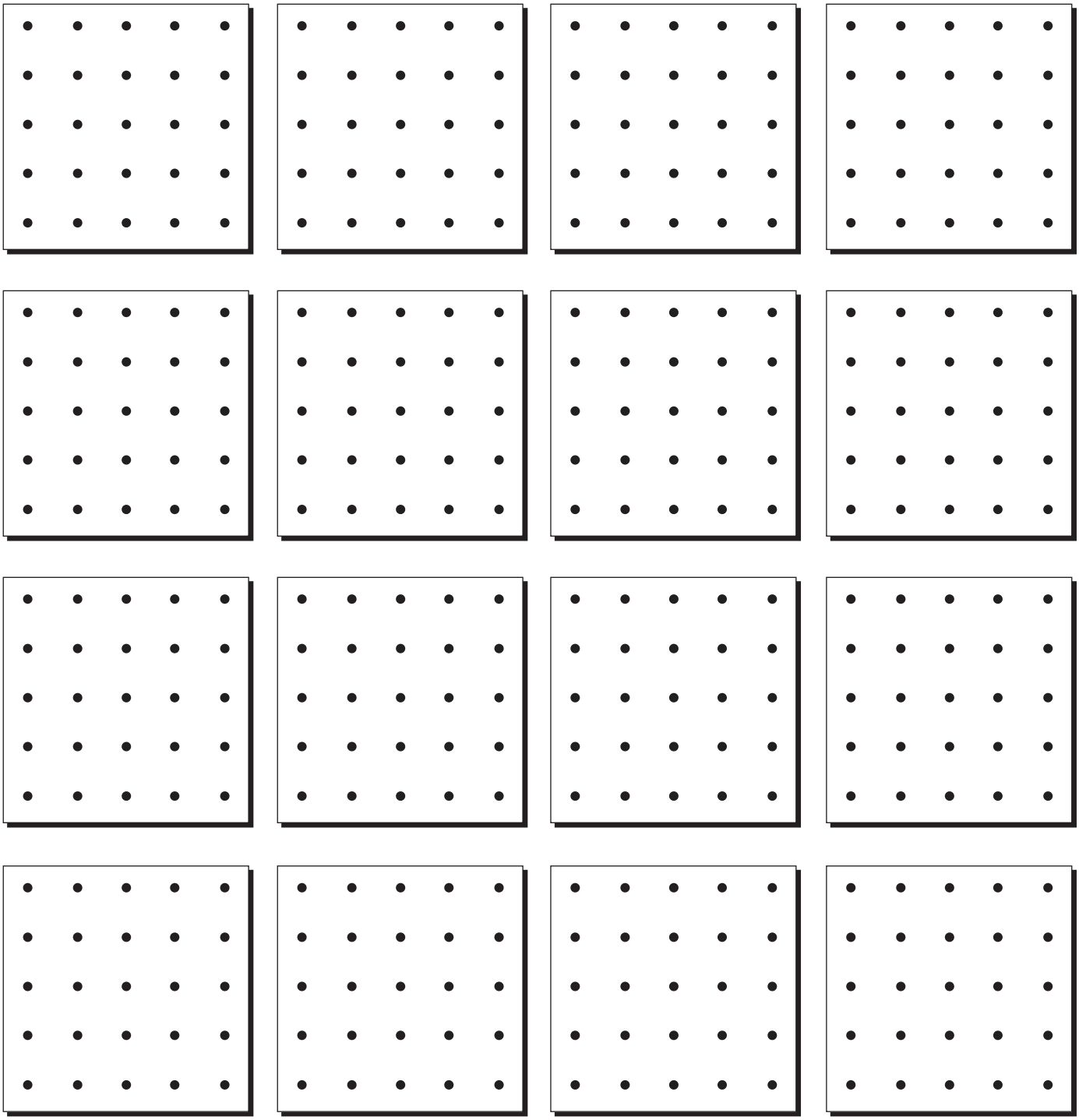
3 List some reasons why you think a base ten counting system is used around the world.

4 In her notebook, Alyssa wrote that a collection of 95 total units was 235_{strnla}. The base was not readable. What do you think was the base? Explain your method for finding it.

Follow-up Student Activity 15.1

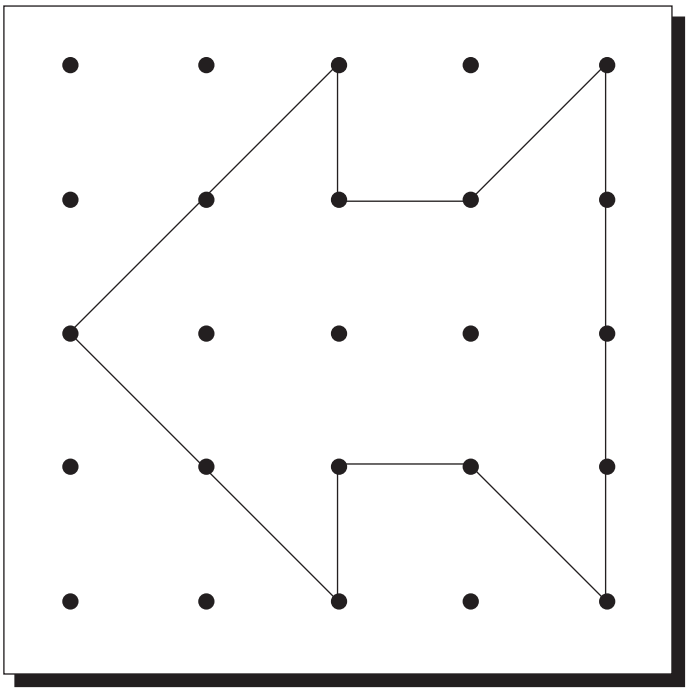
NAME _____ **DATE** _____

1 Find and record polygons with differing numbers of sides; 13, 14, 15, 16, etc. What is the greatest number of sides possible on a 25-pin geoboard?

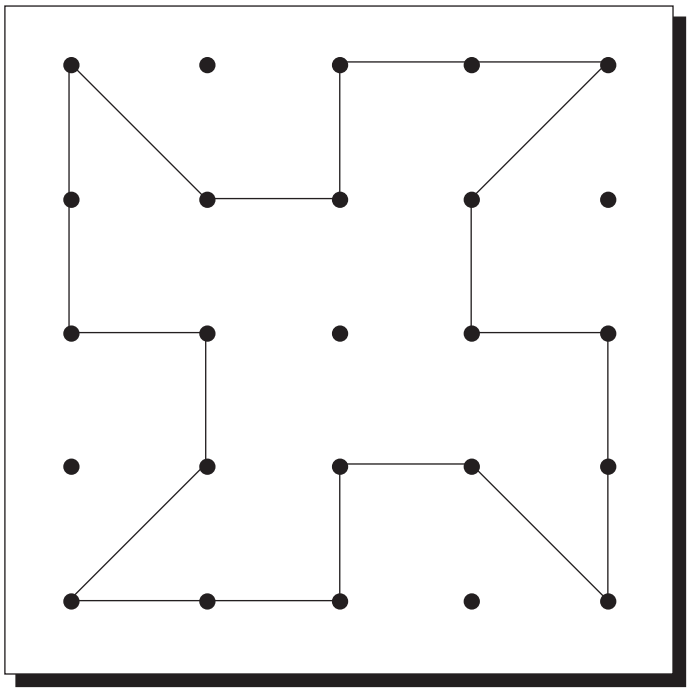


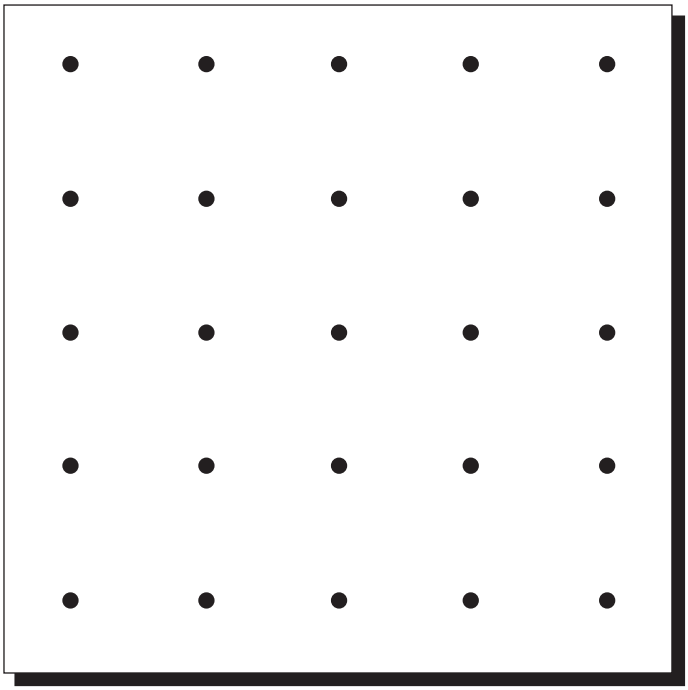
Focus Master A

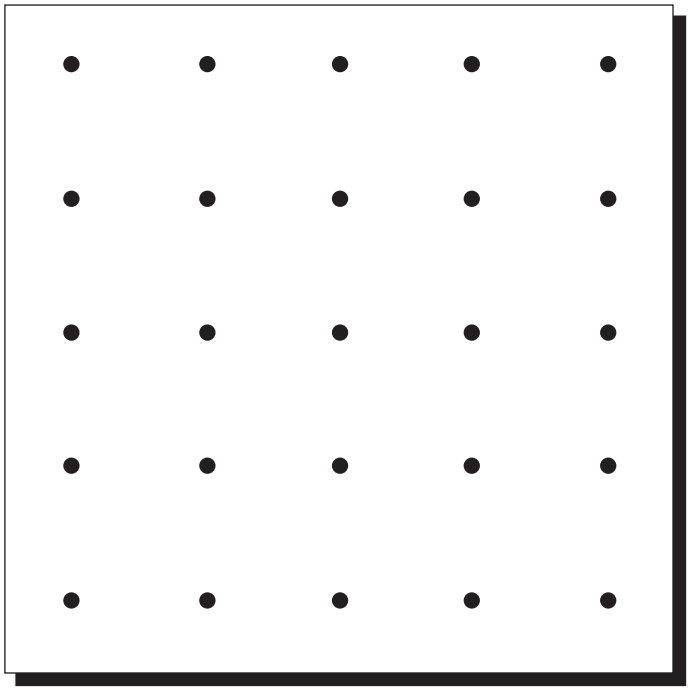
a)



b)







Follow-up Student Activity 16.1

NAME _____ **DATE** _____

1 On the attached dot paper, draw each of the following figures. Be sure to label each figure.

a) This figure has reflective symmetry across a vertical (↕) line *and* a horizontal (↔) line. (Draw the lines of reflection on your figure.)

b) This figure has rotational symmetry of order 2. (Place an "x" on the center of the rotation.)

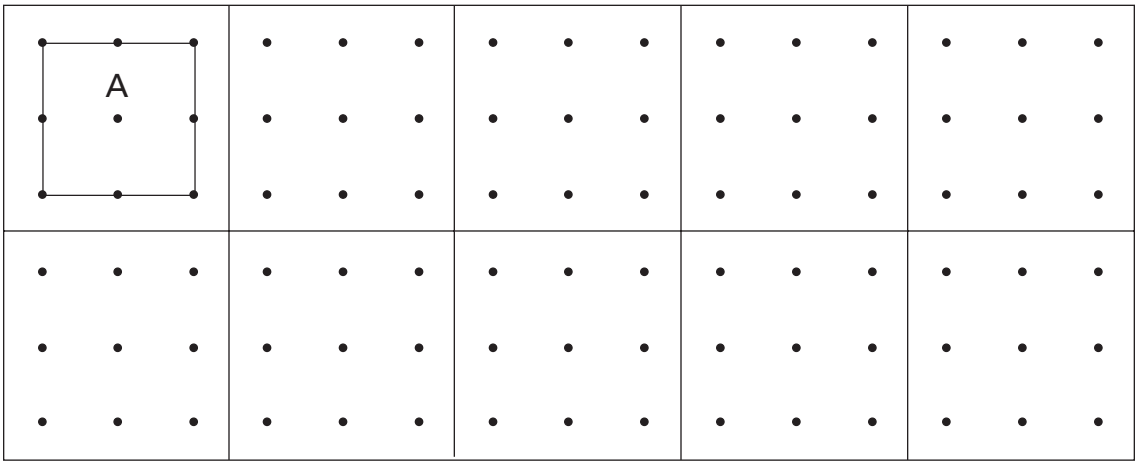
c) This figure is divided into 6 congruent regions.

d) This figure is not a parallelogram. Its area is 12 square units. It has half-turn rotational symmetry. (Place an "x" on the center of the rotation.)

e) This figure does not have reflective symmetry. It is not a simple polygon. Its area is 6 square units. Its perimeter is 22 linear units.

f) This hexagon does not have reflective symmetry. Its area is 4½ square units.

2 Show all the different possible ways to divide Square A into 2 *non*-congruent regions if lines have to connect pin to pin.



(Continued on back.)

Follow-up Student Activity (cont.)

3 Add 1 or more squares to each figure at the right so the statement is true.

a) This figure has exactly 1 line of symmetry.

b) This figure has exactly 2 lines of symmetry.

c) This figure has half-turn rotational symmetry.

4 Write a letter to students in another school to help them understand the meaning of rotational and reflective symmetry.

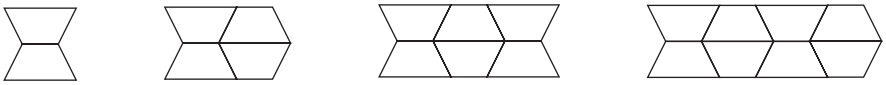
5 Color a different design on each of the attached grids so that each 10 × 10 grid has reflective and/or rotational symmetry. Cut apart the grids and label the back of each colored grid to identify the type of symmetry it has.

6 Cut out pictures of objects, fabric designs, flags, business logos, etc., that have reflective and/or rotational symmetry. Label each picture according to its type of symmetry. Clip your pictures to this assignment.

Follow-up Student Activity 17.1

NAME _____ **DATE** _____

1 The following are the first 4 trains in a sequence.



a) Use your pattern block pieces to form what you think is the 5th train in this sequence. In the space below describe what your 5th train looks like and tell how you decided this.

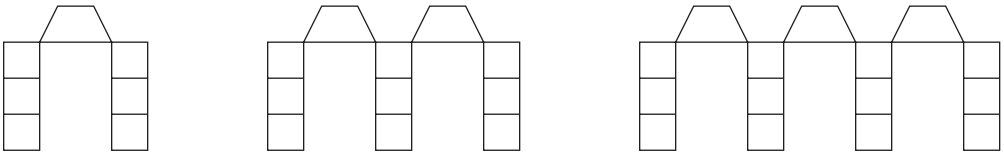
b) Write a description of what you think the 20th train looks like so a person who reads your description could accurately build that train.

c) Which train do you think has 286 trapezoids? Tell how you decided this.

(Continued on back.)

Follow-up Student Activity (cont.)

2 The first 3 trains in a sequence are shown below:



a) Tell how the first 3 trains help you to decide what the 20th train looks like without having to build all of the intervening trains.

b) Write a description of the 100th train in the sequence so a person who reads your description could accurately build that train.

c) If a certain train in this sequence has 45 squares, how many trapezoids are in the train? Explain how you decided this.

d) Show this pattern to an adult. How did they "see" the 100th train and what was their reasoning?

3 Invent an interesting pattern of trains. On another sheet glue or tape in place the first 3 trains in your pattern. On the back of that sheet explain how someone could use your first 3 trains to tell what your 50th train looks like.

Follow-up Student Activity 18.1

NAME _____ **DATE** _____

1 In your own words, explain the meaning of perimeter.

2 The first 3 trains in a sequence are shown below. Questions a)-d) refer to this sequence:

a) If this pattern continues, describe what the 10th train looks like and how you decided this.

b) What is the perimeter of the 10th train, assuming the length of one edge of a small parallelogram is 1 linear unit? Explain how you "see" the perimeter of train 10:

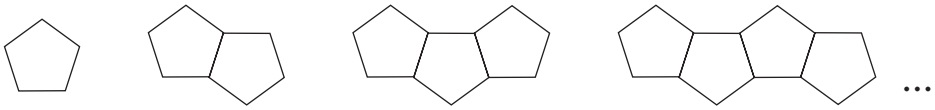
c) Explain how you "see" the perimeter of the 100th train.

d) Which train has a perimeter of 46? Explain how you determined this.

(Continued on back.)

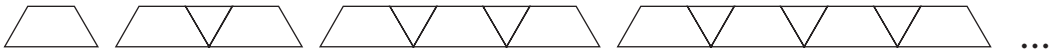
Follow-up Student Activity (cont.)

3 The first 4 trains in a sequence are shown below.



Find the perimeter of the 20th train and explain 2 different ways to "see" this value.

4 The first 4 trains in a sequence are shown below.



Describe a method (other than building the train and counting) that works for finding the perimeter of every train in this sequence.

5 Describe what is easiest and what is hardest now for you about working with visual patterns.

6 Describe one or more ideas about visual patterns that used to be hard for you, but are easier now. Tell what you think helped you to understand better.

7 Use your pattern blocks to form the pattern in problem 4 and show it to an adult. Find out their ideas about what the 100th train looks like and what its perimeter would be. How do their ideas compare to yours? Explain on another sheet of paper.

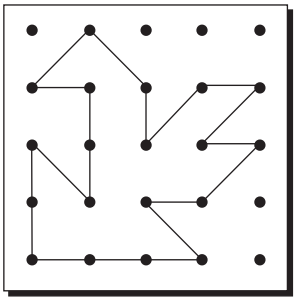
Follow-up Student Activity 19.1

NAME _____ **DATE** _____

1 On the attached sheet of Geoboard Recording Paper, record as many figures as you can that have area 4 square units.

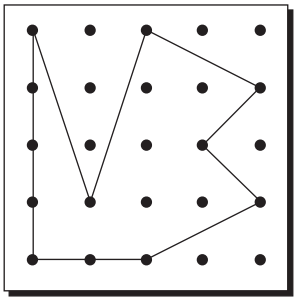
2 Find the area of each of the following figures. Below each figure write an explanation of the methods you used to determine the area and tell what you used as the area unit.

a)



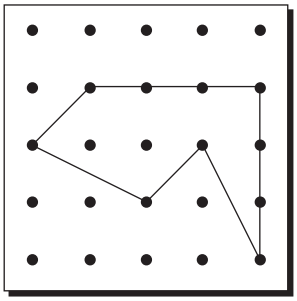
Methods:

b)



Methods:

c)



Methods:

3 Explain in your own words what area means.

(Continued on back.)

Follow-up Student Activity (cont.)

4 If possible, sketch and label these figures on Geoboard Recording Paper. If you think it's not possible write "impossible." (Let 1 small square be the area unit.)

a) Two shapes whose areas are the same, but their perimeters are different.

b) Two figures whose perimeters are the same but whose areas are different.

c) A rectangle whose area and perimeter are the same number.

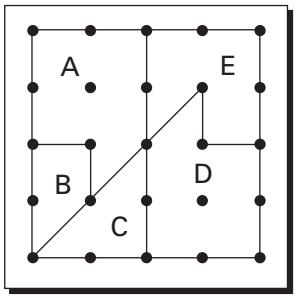
d) Two shapes that are not congruent but have the same area.

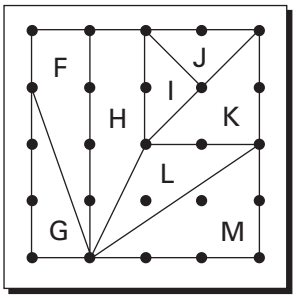
e) Two shapes that are congruent but have different areas.

f) A shape that has 1 line of symmetry which divides the shape into 2 non-congruent parts.

g) A shape that has no line of symmetry but can be divided into 2 congruent parts by 1 line.

5 Suppose the large geoboard square is the area unit. Determine the area of each of the regions A-M below.





Follow-up Student Activity 20.1

NAME _____ **DATE** _____

1 Draw diagrams that illustrate the relationships described below. Besides the information that is given, what are some mathematical conclusions you can make by studying your diagram?

a) The difference between 2 secret numbers is 3.

b) There are 3 secret numbers. The second number is equal to the sum of 2 of the first number. The third number is 5 more than the second number.

c) The length of a rectangle is 7 units more than 2 times its width.

d) The difference between 2 secret numbers is 3. The sum of the 2 numbers is 29.

e) The average of 4 secret numbers is 20. Three of the numbers are equal to each other and the fourth number is 8 more than the sum of the other 3 numbers.

(Continued on back.)

Follow-up Student Activity (cont.)

2 On a separate sheet of paper, show how to use a diagram or model to solve each of these puzzle problems.

a) To drive from town A to town D, one must drive through town B and then town C. It is 10 miles farther from town A to town B than it is from town B to town C. And, it is 10 miles farther from town B to town C than it is from town C to town D. It is 390 miles from town A to town D. How far is it from town A to town B?

b) Al earned $80 baby-sitting last month. He spent $\frac{1}{2}$ of his earnings on clothes, $\frac{1}{4}$ on repairs for his bike, and $\frac{1}{8}$ on movie tickets. He put the rest in savings. How much did he save last month?

c) The average of a set of 5 numbers is $28\frac{1}{2}$. Four of the numbers are 17, 46, 19, and 27. What is the fifth number?

d) Bob has 4 more dimes than nickels. He has $1.45 in all. How many nickels does he have?

e) Find the 2 secret numbers. The second number is equal to $2\frac{1}{2}$ of the first number. Their sum is 84.

f) A rectangle has perimeter 60 inches. The length of the rectangle is 2 inches more than 3 times its width. What are the dimensions of the rectangle?

g) The average of 5 numbers is 11. The second number is equal to the first. The third number is double the first. The fourth number is equal to the first 3 numbers added together. The fifth number is 7. What are the other 4 numbers?

h) A triangle has perimeter 35 inches. The length of side A of the triangle is twice the length of side B. Side C is 5 inches longer than side A. What is the length of each side of the triangle?

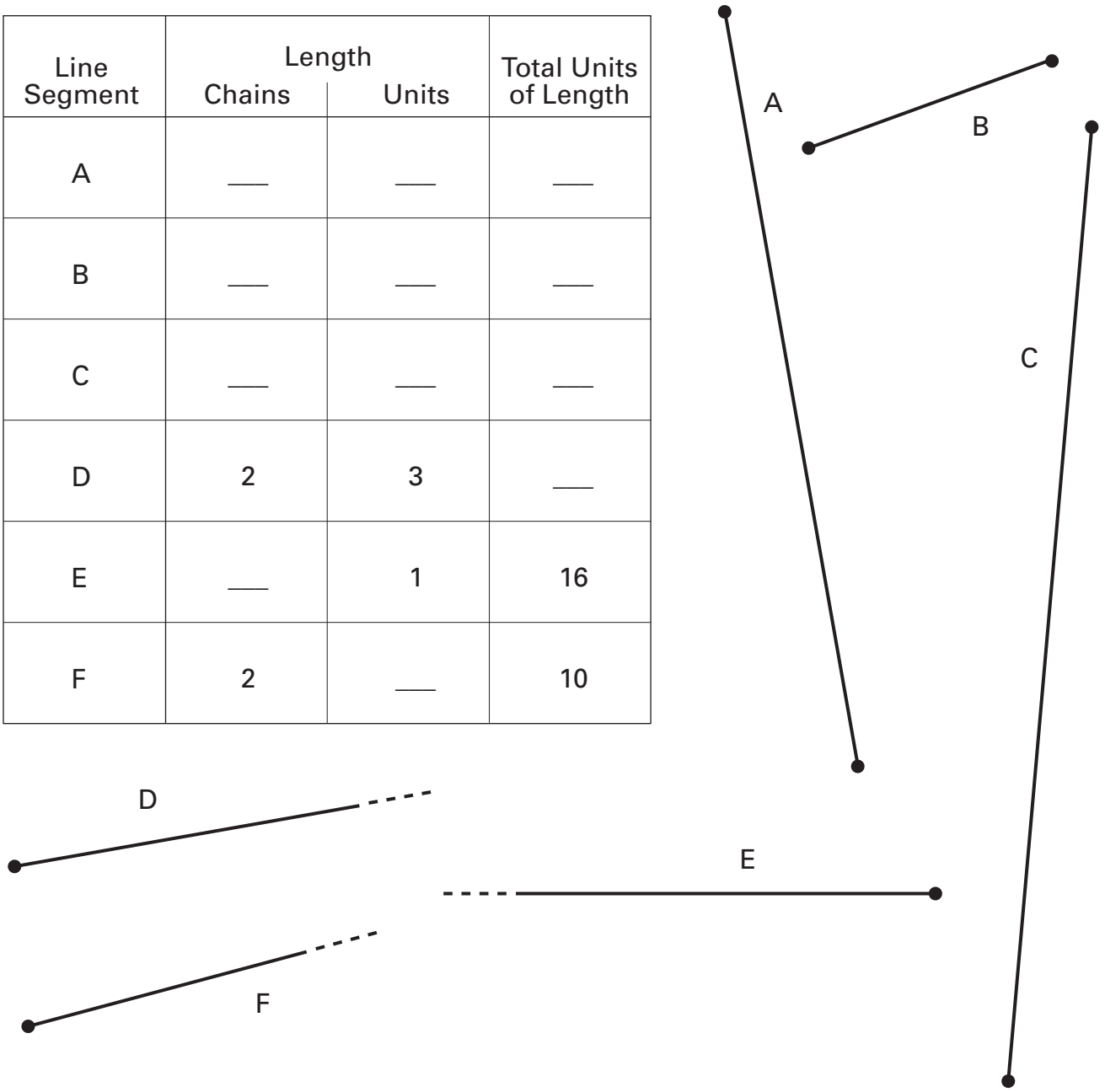
Focus Student Activity 21.1

NAME _____ **DATE** _____

1 Complete the table.

2 Finish drawing segments D, E, and F so their lengths are those given in the table.

Line Segment	Length		Total Units of Length
	Chains	Units	
A	___	___	___
B	___	___	___
C	___	___	___
D	2	3	___
E	___	1	16
F	2	___	10

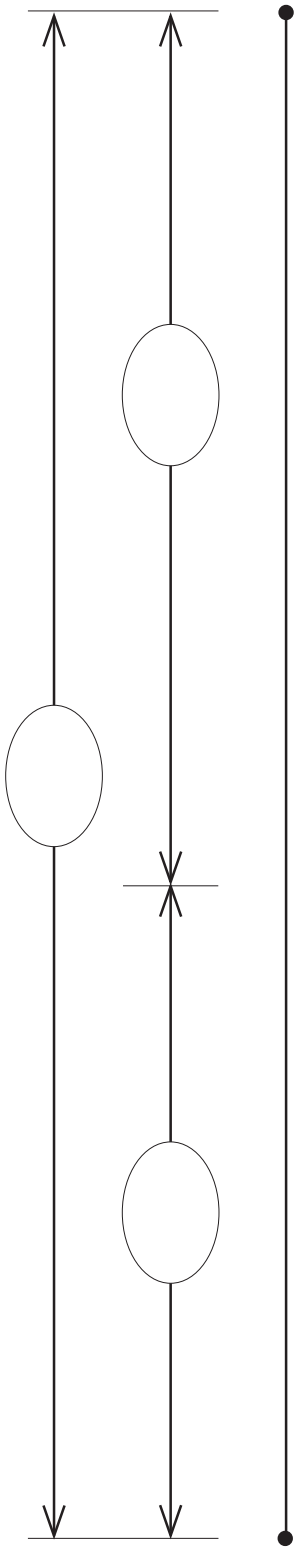


Focus Student Activity 21.2

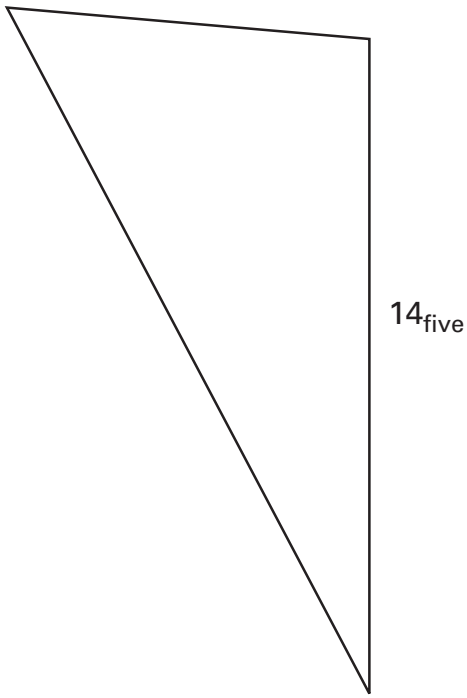
NAME _____ **DATE** _____

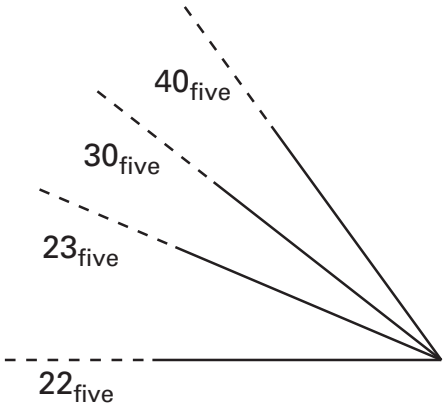
Record the length of this line segment using base five notation.

Fill in the missing lengths using base five notation.



Measure the other 2 sides of this triangle and record their lengths.





Complete the line segments so they have the indicated length

Follow-up Student Activity 21.3

NAME _____ **DATE** _____

1 Cut out the base four ruler on the bottom of this sheet. Use it to measure the dimensions of the items listed below to the nearest linear unit. Then use base four notation to record the dimensions of each item.

a) this sheet of paper

b) the front cover of a book

c) an envelope

d) another item of your choice: _____

2 Use the base four ruler to help draw line segments whose lengths are given below. Draw the segments in the space provided.

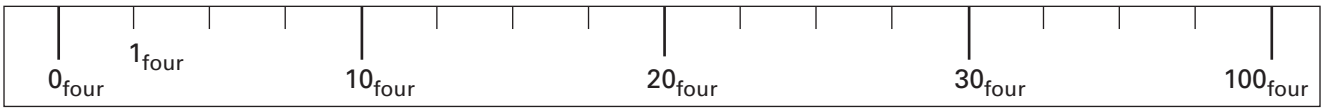
a) 19 linear units

b) 12 linear units

c) 12_{four} linear units

d) 23_{four} linear units

e) 31_{four} linear units



Base four ruler to cut out.

(Continued on back.)

Follow-up Student Activity (cont.)

3 Draw what you think a base seven linear unit and chain look like. Explain how you would draw the length 235_{seven}.

4 For this problem, one linear unit looks like this:

The segment below has length $13_?$.

What do you think is the base in this example? Explain how you can be sure.

5 Jorge changed the linear unit on the base four ruler from Problem 2 to the following.

1_{four} linear unit

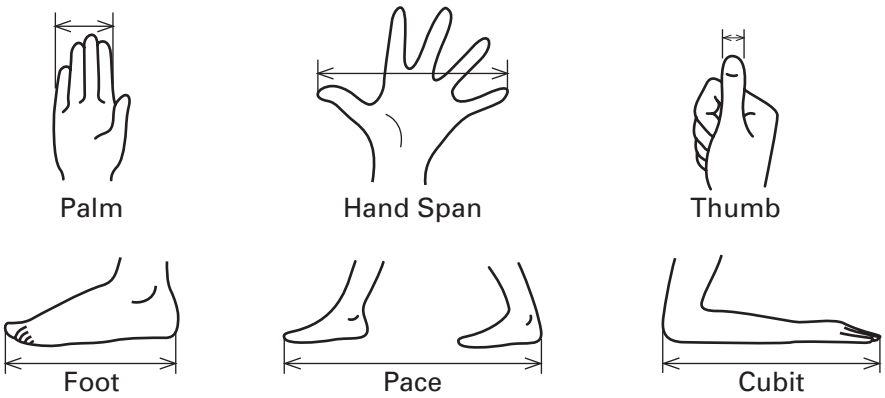
a) Jorge drew the following segment and said its length is $0{,}21_{four}$ linear units (based on the above unit). Explain what you think he meant by this measurement.

b) Explain how you think Jorge would draw the length $1{,}123_{four}$ linear units (using the above unit).

Connector Student Activity 22.1

NAME _____ **DATE** _____

Shown below are several very old units of linear measure.



1 Select 6 objects to measure using your body measures for the above units. Record your measures on the chart below.

Object	Unit Length	Measurement
	Hand span	
	Palm	
	Thumb	
	Cubit	
	Pace	
	Foot	

2 Tell which of the above historical units seems most reasonable for measuring the length of each of these objects. Tell why you selected each unit.

a) the height of a person

c) the thickness of a book

b) the length of a football field

d) the length of a table

(Continued on back.)

Connector Student Activity (cont.)

3 Explain what you think are the advantages and the drawbacks of the historical measures mentioned on page 1.

4 Suppose a ream of paper (500 sheets) is 3 thumb widths thick. How thick (in thumb widths) is one sheet? Explain your methods.

5 Each of these segments was drawn using a different linear unit. Without using a ruler, invent a method of locating and labeling a close approximation of 1, 2, and 3 on each segment. (You will need to extend some segments.)

a)

b)

c)

d)

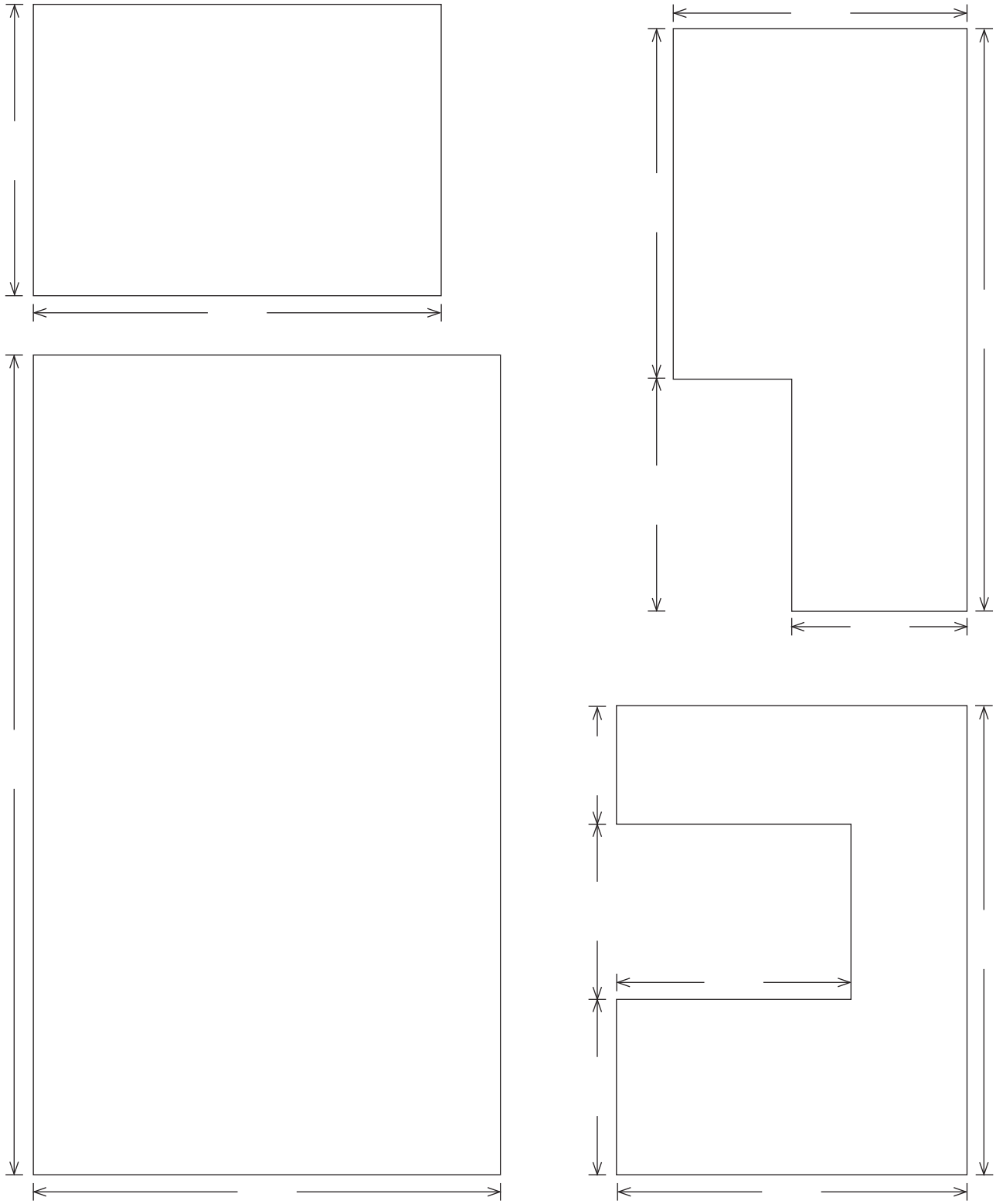
e)

f) On another sheet, explain the methods you used for b), d), and e).

Focus Student Activity 22.2

NAME _____ **DATE** _____

Write the areas and dimensions in base five notation.



Follow-up Student Activity 22.3

NAME _____ **DATE** _____

1 Use base five number pieces to construct each of the following rectangles and then record the information requested using base five notation.

a) A rectangle whose area is 341_{five}. Record its dimensions.

b) A different rectangle whose area is 341_{five}. Record its dimensions.

c) A square whose side has length 21_{five}. Record its area.

d) A rectangle with dimensions 11_{five} and 23_{five}. Record its area.

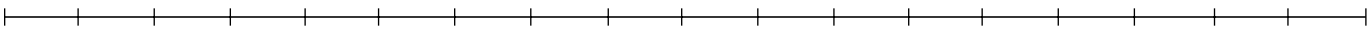
e) A rectangle with area 223_{five} and one dimension 14_{five}. Record its other dimension.

f) A rectangle with area 312_{five} and one dimension 10_{five}. Record its other dimension.

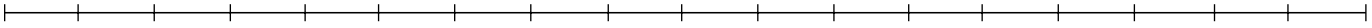
2 Explain how a linear unit and an area unit are alike and how they are different.

3 In problems a)-d) that follow, ⊢——⊣ is 1 linear unit. On each of the line segments mark off the indicated length and record the total number of linear units it contains.

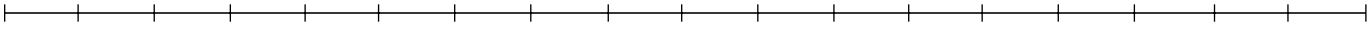
a) 13_{five}



b) 111_{three}



c) 14_{twelve}



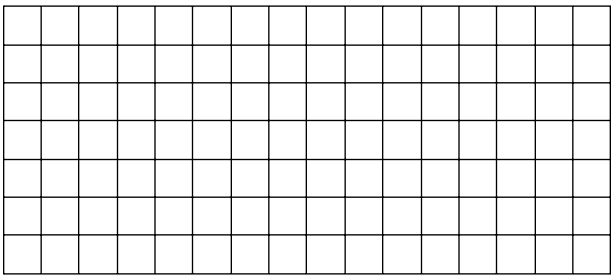
d) 14_{ten}

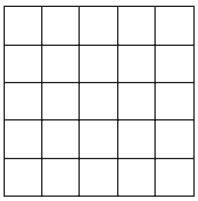
(Continued on back.)

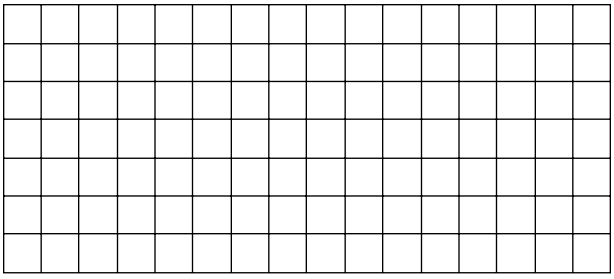
Follow-up Student Activity (cont.)

4 Mark each rectangle below to show how you determined its area and dimensions.

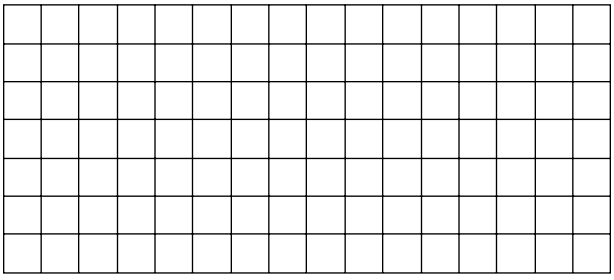
a) Suppose that ☐ is 1 area unit and ⊢⊣ is 1 linear unit, what are the base five dimensions and area of this rectangle?



b) Suppose that  is 1 area unit and  is 1 linear unit. Now what are the base five dimensions and area of this rectangle? Explain how you determined these measurements.



c) Suppose that ⊢⊣ is 1 linear unit and ☐ is 1 area unit. What are the base four dimensions and area of this rectangle? Explain how you determined these measurements.



Follow-up Student Activity 23.1

NAME _____ **DATE** _____

1 Explain in your own words the meaning of the *take-away* model for subtraction. Draw a diagram to show an example.

2 Explain in your own words the meaning of the *difference* model for subtraction. Draw a diagram to show an example.

3 Explain and draw a picture to illustrate what addition means to you.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Use your base five pieces to do the following computations. Then make a sketch to show your methods with the pieces. Be sure to show how you traded pieces. Next to each sketch, write the answer using base five notation.

a) $324_{five} + 441_{five}$

b) $432_{five} - 243_{five}$

5 Draw a sketch of base eight area pieces to show how to use them to find $135_{eight} + 257_{eight}$. Show any trades you need to make.

6 Draw a sketch of base three linear pieces and show how to use them to find $121_{three} - 22_{three}$. Show any trades you make.

Follow-up Student Activity 24.1

NAME _____ **DATE** _____

1 Explain and draw diagrams to show different ways to view the meaning of multiplication.

2 Explain and draw diagrams to show three different ways to view the meaning of division.

3 Use base five pieces to compute $1313_{\text{five}} \div 23_{\text{five}}$. In the space that follows, explain your methods and then use base five notation to record the quotient.

4 Use your base five pieces to compute $32_{\text{five}} \times 14_{\text{five}}$. In the space below, explain your methods and use base five notation to record the product.

(Continued on back.)

Follow-up Student Activity (cont.)

5 Make sketches on the attached base five grid paper to carry out the following computations. Label your sketches and record the answers using base five notation. (It may be helpful to use base five pieces first and then record your base five piece methods on the grid paper.)

a) $34_{five} \times 23_{five}$

b) $231_{five} \div 11_{five}$

c) $324_{five} + 143_{five}$

d) $431_{five} - 143_{five}$

e) $24_{five} \times 13_{five}$

f) $1013_{five} \div 34_{five}$

6 On the grid below sketch diagrams to show how to use the area model to compute: $24_{six} \times 13_{six}$ and $2102_{three} \div 12_{three}$. Label your sketches.

Follow-up Student Activity 25.2

NAME _____ **DATE** _____

1 Draw each of the following on base ten grid paper. Label each drawing to show the base ten number it represents.

a) ☐ is one square unit. Draw the minimal collection for 32 square units.

b) ⊢⊣ is one linear unit. Draw the minimal collection for 12 linear units.

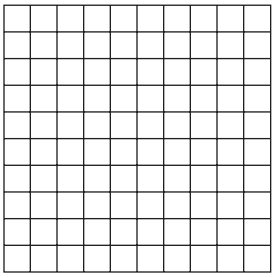
c) ☐☐☐☐☐☐☐☐☐☐ is one tenth. Draw 2 units.

d) ☐☐☐☐☐☐☐☐☐☐ is one unit. Draw 2.3 units.

e) ⊔⊔⊔⊔⊔⊔⊔⊔⊔⊔ is 1/10. Draw 1/100.

f) ⊔⊔⊔⊔⊔⊔⊔⊔⊔⊔ is 1. Draw 3.7.

g) ⊔⊔⊔⊔⊔⊔⊔⊔⊔⊔ is 1. Draw a length that is between .2 and .3 linear units (be sure to write the number your drawing represents).

2  is one square unit. Draw and label the following on base ten grid paper:

a) 1.2
b) 1.02
c) 1.20
d) 0.12
e) 10.2
f) 2
g) 2.1
h) 1.002

3 Imagine or sketch the minimal collection for each number below. Then imagine trading all the pieces in each collection for *hundredths only*. Write how many total hundredths there are in each collection.

a) 4.13 b) 8.9 c) 2.07

(Continued on back.)

Follow-up Student Activity (cont.)

d) Explain how you found the total hundredths in b).

4 In this space sketch a minimal number piece collection for 3.247.

5 Imagine exchanging all the number pieces in Problem 4 for thousandths. Describe the results.

6 Imagine exchanging all the pieces in Problem 4 for hundredths. Describe the results.

7 Imagine (or sketch) the number piece collections for the following numbers. Label the unit you use for each collection. Then write in words *two different ways* that you could read each number if you were talking to someone over the telephone.

a) 1.27

b) 72.04

c) 127.654

8 The odometer of a car reads 35467.21. The digit 5 stands for 5000 miles. Tell what each of the other digits stands for:

a) 1

b) 2

c) 3

d) 4

e) 6

f) 7

Follow-up Student Activity 26.2

NAME _____ **DATE** _____

1 Please read the letter on the back of this sheet. Then fill in the blank after *Dear* to show the name of the adult you would like to teach about your thinking. Sign your name in the space at the bottom of the letter.

a) Have the adult read the letter *before* you do your presentation. Plan to spend about 30 minutes on this assignment.

b) *After* your presentation, complete the following:

What I did was…

I feel very good about the way I presented…

because…

Something that was difficult or challenging for me during this presentation was…

Tomorrow I plan to focus my attention during class discussion on the following ideas about decimals so that I can understand or explain them better:

(Continued on back.)

Follow-up Student Activity (cont.)

Dear _____,

In math right now, we are learning about addition and subtraction of decimals. Instead of memorizing the results of someone else's thinking, we are going to use our understanding of the meaning of addition and subtraction and our understanding of the meaning of decimals to do the work of real mathematicians—beginning tomorrow we will *invent* a variety of systems and rules for operating with decimals! Please keep the rules you know for adding and subtracting decimals a secret for now—it will be fun to compare later. Since explaining my thoughts and ideas helps me to know what I understand and what I have questions about, would you take about 30 minutes to listen while I explain my thinking about *at least four* of the following:

a) how the base ten pieces are related to each other and what even bigger and smaller pieces look like (and why I think this is so);

b) what happens to the value of pieces when I "change the unit" and why this happens (I'll give you some examples);

c) what addition and subtraction mean to me;

d) how I can use the base ten pieces to add 252 + 149 and then to add 3.47 + 1.67 (keep me talking about my thinking while I work!);

e) how I can use the base ten pieces to subtract 232 – 174 and 4.06 – 2.38;

f) how I can use mental pictures of the base ten pieces to mentally compute 3.99 + 4.26 and 5.3 – 2.7.

It is okay if I get stuck. One important reason for sharing my thinking with you is so that I can know where to focus my attention and ask questions during class. After I finish, would you help me by reminding me to write down ideas that were difficult for me to explain? Thanks for listening to my thinking!

_____

Student's signature

Please write down two aspects of my presentation you think I should be pleased about. Then sign your name.

_____

Adult Listener's signature

Follow-up Student Activity 27.1

NAME _____ **DATE** _____

1 Use your area pieces to determine 3.24 + 2.79. Then write, *in words only*, a careful explanation of each step of your area piece methods.

2 Invent a way to use *numbers and arithmetic symbols only* to tell the step-by-step story of how you used area pieces to find 3.24 + 2.79.

3 Use area pieces to determine 3.372 – 1.581.

a) Sketch your area piece methods (show all trades):

b) Describe in words each step of your area piece methods:

c) Invent a way to use numbers and arithmetic symbols only to tell the step-by-step story of how you used your area pieces to find 3.372 – 1.581.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Explain three different ways to compute 2.74 + 4.26 mentally. Circle the method you like best.

5 Explain three different ways to compute 15.3 – 9.8 mentally. Circle the method you like best.

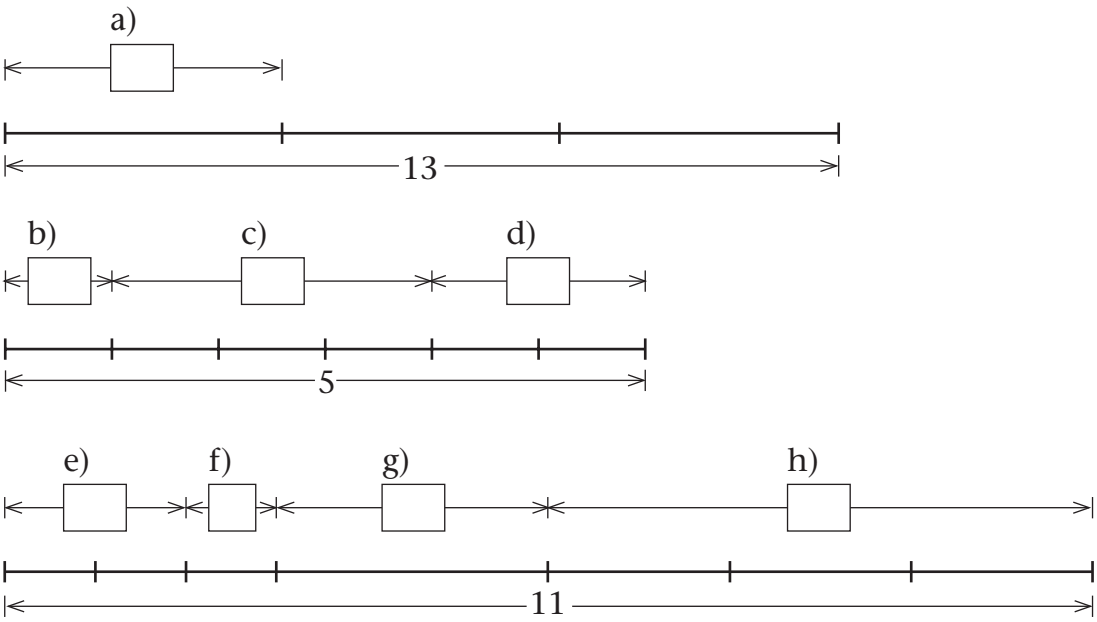
6 Explain in your own words what the word *algorithm* means.

7 When do you think algorithms are helpful and when aren't they helpful?

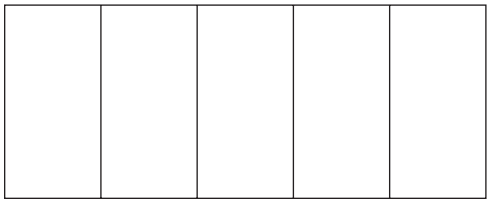
Focus Student Activity 28.1

NAME _____ **DATE** _____

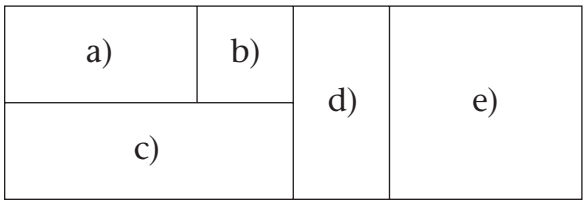
1 Write in the missing lengths in the boxes.



2 This field contains 3 acres. How many acres are in each part?



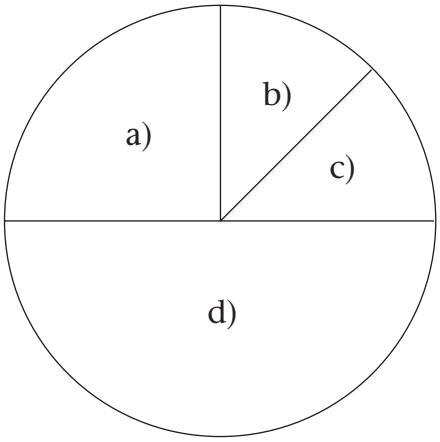
3 This rectangle has area 15. What is the area of each part?



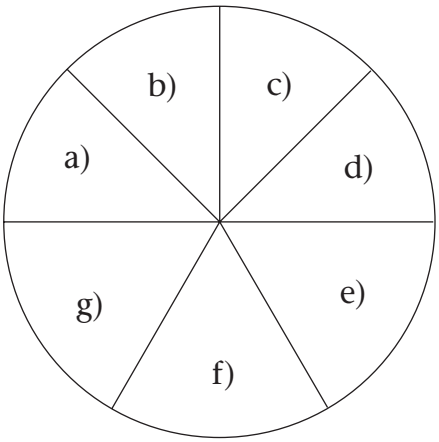
(Continued on back.)

Focus Student Activity 28.1 (cont.)

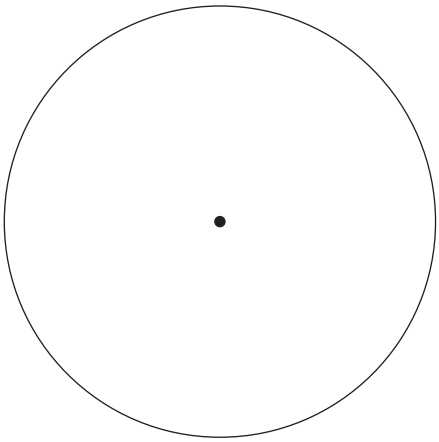
4 The area of this circle is 11. What is the area of each part?



5 This circle has an area of 7. What is the area of each part?



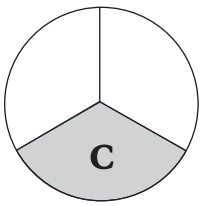
6 This circle has area 12. Subdivide it into parts with these areas: 6, $12 \div 4$, $^{12}/_{12}$, and $12 \div 6$.



Follow-up Student Activity 28.2

NAME _____ **DATE** _____

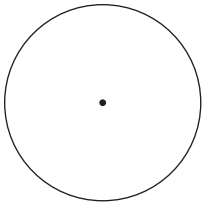
1 The area of region C is $5/3$ square units. What is the area of the circle?



2 The area of regon D is 3 square units. Mark off a rectangle with area 5 square units.

Write an explanation of your method of locating 5 square units.

3 The area of this circle is 5 square units. Shade a part of the circle which has an area of $5/4$ square units. Next to the circle, explain your methods of determining the part of the circle with area $5/4$ square units.



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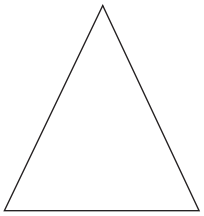
Follow-up Student Activity (cont.)

4 The area of this triangle is 3 square units. Shade a part of the triangle which has an area of $3/2$ square units.



5 The area of F is $4/3$ square units. Mark off a rectangle with an area of 4 square units:

Write an explanation of your method of marking off a rectangle of area 4 square units:

6 On a separate sheet of paper, show how to use a diagram or model to solve these puzzle problems:

a) During the last 4 weeks Michelle has done yard work after school for her neighbor. Her weekly earnings have been \$9, \$15, \$13, and \$6. What was her average weekly earnings for the 4 weeks?

b) Bill cut a 5-foot piece of rope into 5 "mystery" lengths and gave the following clues about the lengths:

- If the 5-foot piece of rope were cut into 12 equal parts, Mystery Length A has the length of one of those parts.
- Mystery Length B is as long as 3 copies of Mystery Length A placed end-to-end.
- Mystery Length C is half the length of Mystery Length A.
- Mystery Length D is half the length of Mystery Length B.
- Mystery Length E is the remainder of the rope.

How long is each of the mystery lengths?

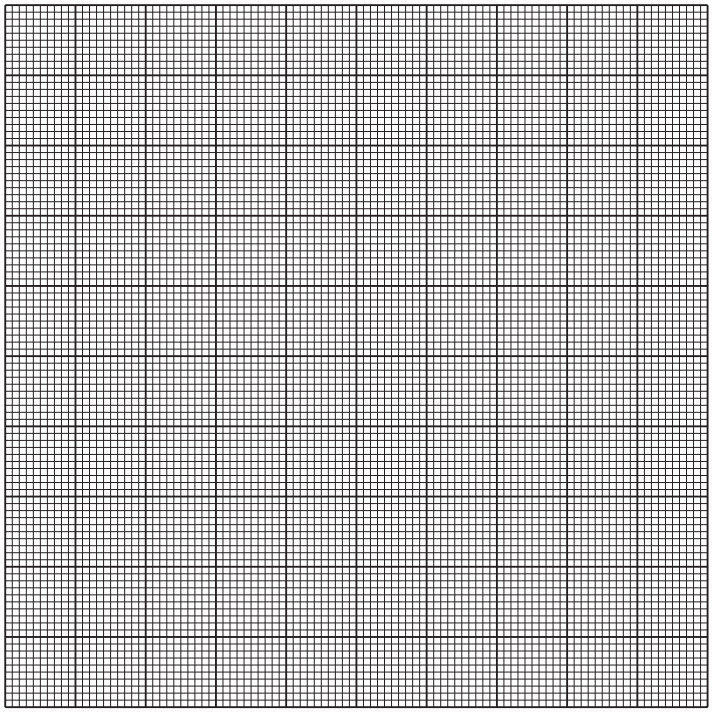
Follow-up Student Activity 29.1

NAME _____ **DATE** _____

1 Form a collection of base ten area pieces that has value 3 ÷ 2. Sketch your collection here. Be sure to label your unit.

2 Write a decimal and a fraction that represent your area piece collection in Problem 1.

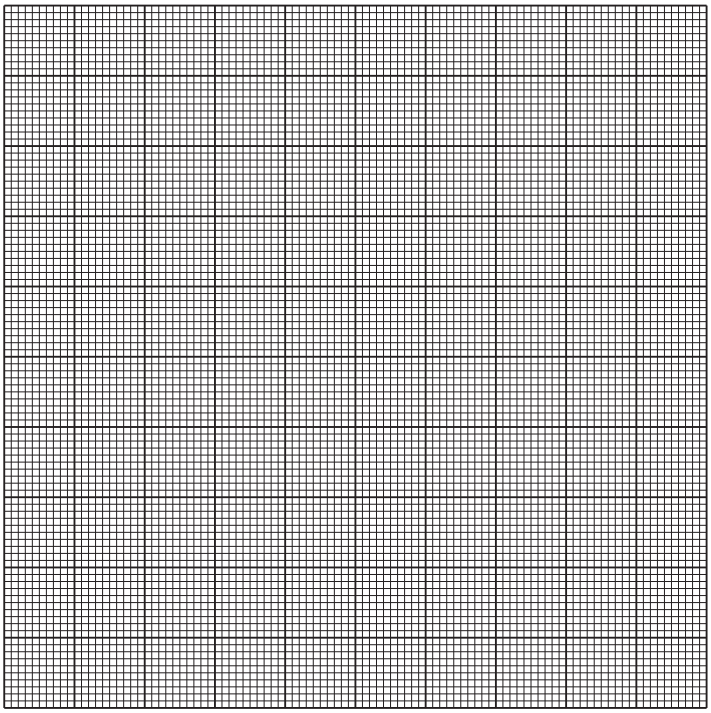
3 Peter says that $\frac{9}{20}$ = .450. On the grid shown below, sketch a "proof" to show why you think he is correct or why he is incorrect. Next to your sketch, write a careful explanation of your proof.



(Continued on back.)

Follow-up Student Activity (cont.)

4 Show how to find the decimal representation for $\frac{1}{6}$ on this grid. Explain your methods.



5 Use a calculator to find 5 fractions that you think represent infinite repeating decimals. Write both the decimal and the fraction form of each number.

6 Use a calculator to find 5 fractions that represent finite terminating decimals. Write both the decimal and the fraction form of each number.

7 Use your calculator to explore decimal representations of a variety of other fractions. On another sheet, explain what you did. What did you notice? Do you have any conjectures? Do you have any questions?

Follow-up Student Activity 30.1

NAME _____ **DATE** _____

1 Each of the following statements is incorrect. Place a decimal point or change the position of the decimal point in each number so that the use of the decimal makes sense.

a) Monica's mother is 54 feet tall.
b) Darrell was paid $235 for baby-sitting for one hour.
c) Eric's father is .400 years old.
d) Mrs. Rogers' newborn baby weighed 92 pounds.
e) The official height of a basketball hoop from the floor is .0100 feet.
f) The Smith's bought a turkey which weighed 120 pounds.
g) The distance across the United States is about .300000 miles.

2 In the space below, sketch number piece collections for each set of numbers. Then use your diagram to order the numbers from smallest to largest.

a) .32 .03 3.02 .30

_____ < _____ < _____ < _____

b) 2.17 .217 .021 .007

_____ < _____ < _____ < _____

(Continued on back.)

Follow-up Student Activity (cont.)

3 Draw a sketch of number pieces that represent 3.849.

Now draw the collection closest in value to 3.849 that uses:

a) only units and tenths b) only units c) no pieces smaller than hundredths

4 Round 596.499 to the nearest:

a) unit _____ c) tenth _____
b) thousand _____ d) hundredth _____

5 Imagine a number line that extends from 0 to 160. Name 3 decimals that are:

a) between 154 and 155 _____, _____, _____
b) between 3.7 and 3.8 but closer to 3.7 _____, _____, _____
c) between 14.62 and 14.63 but closer to 14.63 _____, _____, _____
d) greater than $1/4$ and less than $1/2$ _____, _____, _____

6 Arrange these numbers in order from smallest to largest and then explain your methods.

8.932 7.912 $8\frac{9}{10}$ 9.111 8.99 $7\frac{9}{12}$ 8.24

7 On separate paper, write a letter to a younger student (who has base ten pieces) explaining how to use the base ten pieces to understand rounding decimals.

Focus Master G

Approximation Situations

Describe what you think are reasonable approximation methods to use for each of the following situations. Write your observations and conclusions about each situation.

a) Jan earned $178 one week and $235 the next week. She needs $400 to buy a guitar. She used approximation to decide if she had enough money.

b) Bill's car gets 30 miles per gallon on the highway and he is going to travel 153 miles. He will make a trip on December 23 and the temperature is expected to be below 15°. To insure that he will have enough gasoline to make the trip, he uses approximation to decide how much gasoline to buy.

c) Baseball cards cost $2.47 each and Sandra wants to buy 3 cards. She uses approximation to determine the amount of money she needs.

d) The distance from the ground to the roof of a building is 44 feet. Parker uses approximation to decide whether to buy a 40 foot ladder or a 50 foot ladder to use while repairing the roof of this building.

e) Alesea averaged 17.6 points per game for 4 games. The coach used approximation to estimate whether Alesea had scored more than the record of 67 points in 4 games.

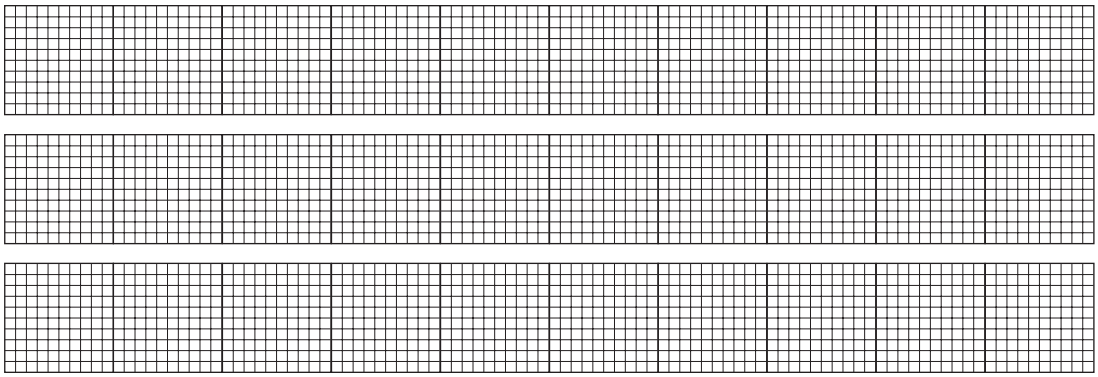
f) In 3 weeks of fund raising the class had earned $139, $259, and $196. The students approximated to determine if they had raised more or less money than their goal.

Follow-up Student Activity 31.1

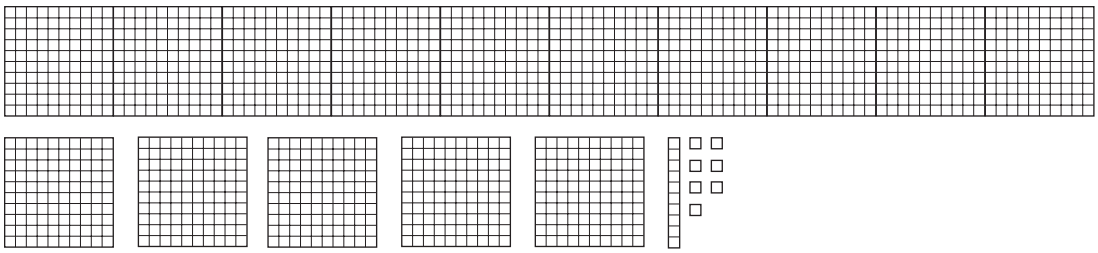
NAME _____ **DATE** _____

1 Describe two different methods of approximating the difference between these two collections of number pieces. (The smaller square " ▫ " is the unit.)

Collection A:



Collection B:



Method 1:

Method 2:

2 Describe two different methods of approximating this sum:
5191 + 2365

Method 1:

Method 2:

(Continued on back.)

Follow-up Student Activity (cont.)

3 Estimate each answer. (Note means "is approximately equal to.") Then explain the methods you used for each problem.

a) 32 + 11 + 23 + 69 + 94 + 76

b) 3.4932 + 27.2097 + 1.403 – 16.909

c) 286.41 – 158.09

4 Use your calculator to solve each computation below.

a) 24.36 + 103 – 47.66 + 75.23 – 149.1 = _____

Use estimation strategies to decide whether the calculator answer is reasonable. Then describe your estimation strategies here:

b) .4689 + .713 + .49 – 1.0099 = _____

Use estimation strategies to decide whether the calculator answer is reasonable. Then describe your estimation strategies here:

c) 197.2013 – 9.96 – 73.6217 + 10.3 + 49.98 = _____

Use estimation strategies to decide whether the calculator answer is reasonable. Then describe your estimation strategies here:

5 On another sheet of paper, explain some techniques for estimating sums and differences that you didn't know before you took this class or that you now understand better.

Follow-up Student Activity 32.2

NAME _____ **DATE** _____

1 If possible, use 2 *non*congruent pattern blocks to form the following angles. If not possible, explain why. On another sheet, trace around each angle and the pattern blocks you use to make the angle. Write the total number of degrees in each angle.

a) a right angle (90°)

b) an obtuse angle (greater than 90° and less than 180°)

c) an acute angle (less than 90°)

d) a straight angle (180°)

e) a reflex angle (greater than 180°)

2 If possible, use pattern blocks to form each of the following angles. If not possible, explain why. On another sheet, trace each angle and the pattern blocks that you use. Write the total number of degrees in each angle.

a) a 360° angle, using 5 noncongruent pattern blocks

b) a 360° angle, using 1 obtuse angle, 1 right angle, and 1 acute angle

c) a 360° angle using 3 different *colors* of pattern blocks and so that the angle formed by color B is 3 times the angle formed by color A, and the angle formed by color C is 8 times the angle covered by color A. (Hint: you can use more than 3 pattern blocks.)

(Continued on back.)

Follow-up Student Activity (cont.)

3 On another sheet of paper, use the pattern blocks to help you draw *approximations* of the following angles. Next to your drawings, explain how you approximated c) and e).

a) 15°

b) 10°

c) 45°

d) 135°

e) 202½°

f) 1°

4 On another sheet, use your pattern blocks to help you draw, if possible, each of the following. If you don't think it's possible explain why. Label the measure of every angle in each shape you draw.

a) A right triangle with a 15° angle.

b) An obtuse triangle (a triangle with an obtuse angle).

c) A right triangle with 1 obtuse angle.

d) A scalene triangle (all sides are different in length) with 2 equal angles.

e) A triangle with angles 45°, 15°, and 135°.

f) A parallelogram with a 60° angle.

g) A parallelogram with a 45° angle.

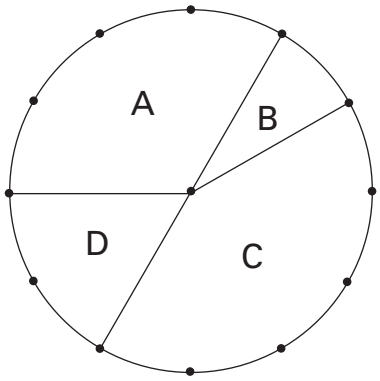
h) A parallelogram with a 60° and a 45° angle.

5 On another sheet of paper write as much as you can to show your understanding of the word *angle*. Include diagrams to illustrate your explanation.

Follow-up Student Activity 33.3

NAME _____ **DATE** _____

1 a) Record what you think is the measure of each central angle on the following circle graph.



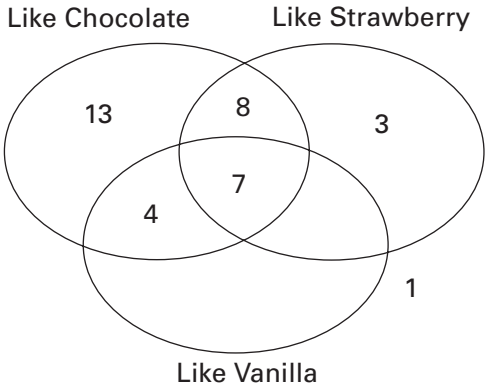
A: _____ B: _____

C: _____ D: _____

b) Tell how you decided the measure of angle A:

c) Make up a situation that could be represented by the above circle graph. Describe the situation here. Give enough details about the situation to show why the regions and angles would have to be the size they are.

2 Here is a Venn Diagram of a survey taken in another class about their opinions of chocolate, strawberry, and vanilla ice cream. In the space below, write ten or more interesting conclusions you can make by studying the mathematical relationships in the graph.

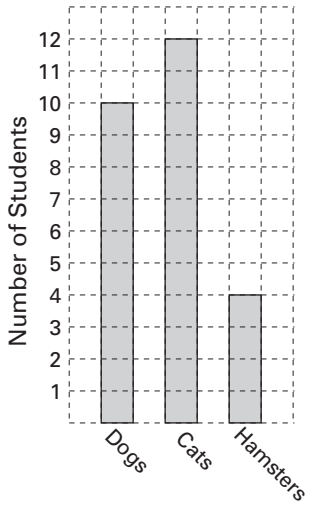


(Continued on back.)

Follow-up Student Activity (cont.)

3 On another sheet of paper make a circle graph, a bar graph, and a rectangular area graph of the data shown in the Venn Diagram in Problem 2. Label your graphs carefully, including approximations of the measure of each angle of your circle graph.

4 Ingrid drew the following graph to show the pets owned by students in her class. Shown at the right of the graph is some other information about the class.



- There are less than 30 students in the class.
- Seven students own both a dog and a cat.
- No one owns all 3 animals.
- No one owns both a hamster and a cat.
- Three students own only a hamster.
- Six students have no pets and no one has a pet other than a dog, cat, or hamster.

How many students are in the class? Explain how you decided this.

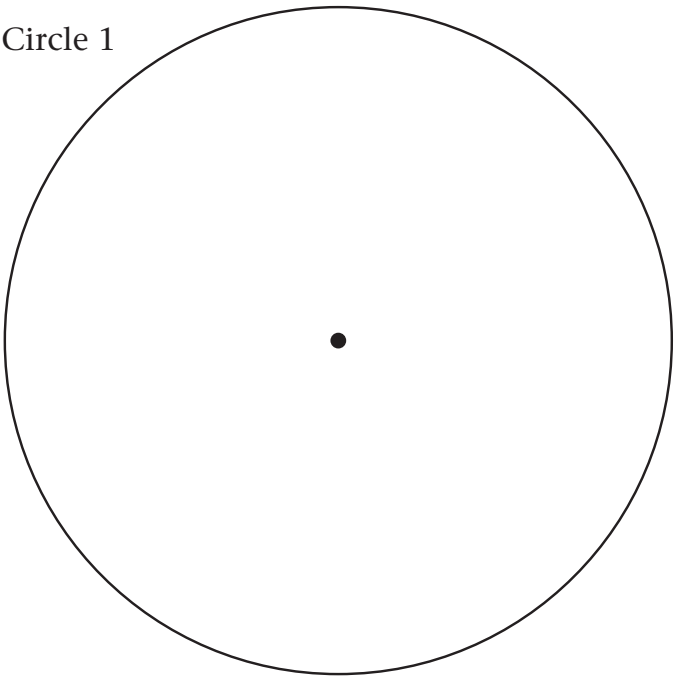
5 Conduct a survey of 20 people regarding their opinions about 3 things (other than ice cream flavors or other topics already surveyed in class). On another sheet, make a bar graph, circle graph, rectangular area graph, and Venn Diagram of your data. Also write 5 important conclusions revealed by the mathematical relationships in your graphs.

Focus Student Activity 34.1

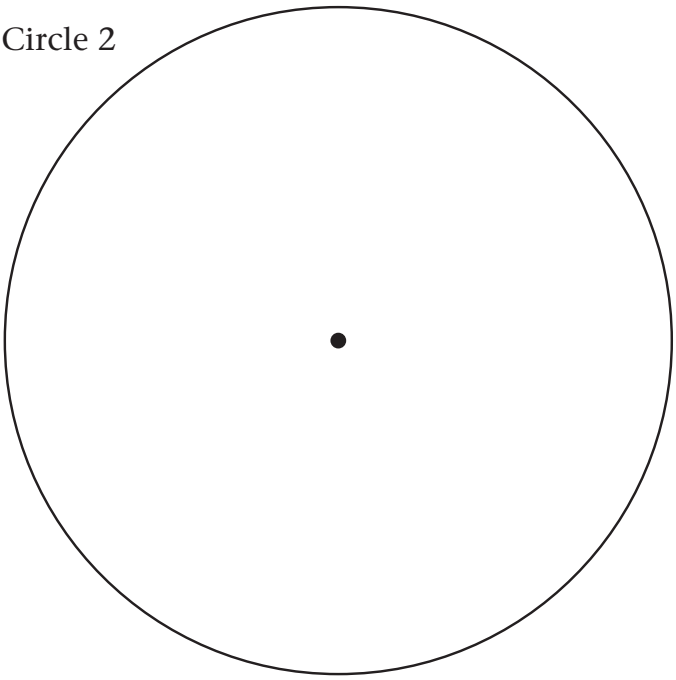
NAME _____ **DATE** _____

- 4 central angles divide Circle 1 into 4 sectors (1 red, 1 blue, 1 green, and 1 orange).
- 1/8 of Circle 1 is blue.
- 1/3 of Circle 1 is red.
- The red sector of Circle 1 covers twice the area that the green sector covers.
- The rest of Circle 1 is orange.

Circle 1



Circle 2



We predict the following as reasonable ranges for the number of times a pointer will land on each color in 40 spins on Circle 1:

Blue: between _____ and _____

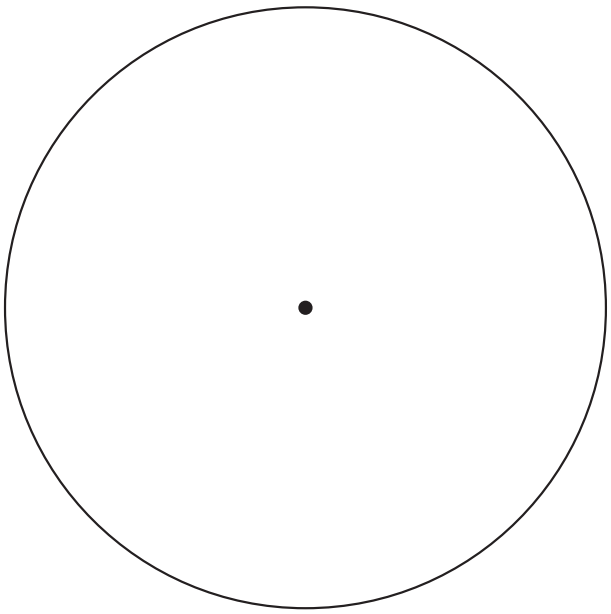
Red: between _____ and _____

Green: between _____ and _____

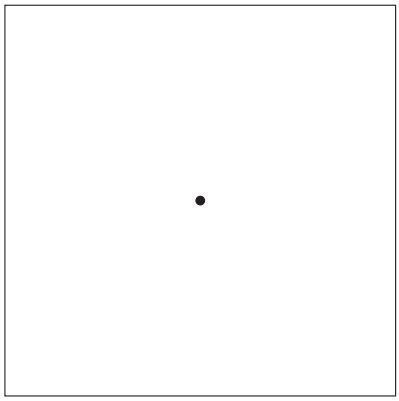
Orange: between _____ and _____

Focus Student Activity 34.2

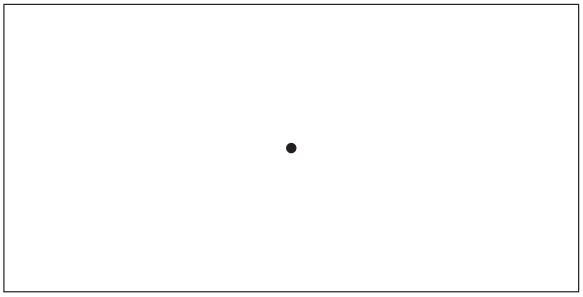
NAME _____ **DATE** _____



Spinner 1

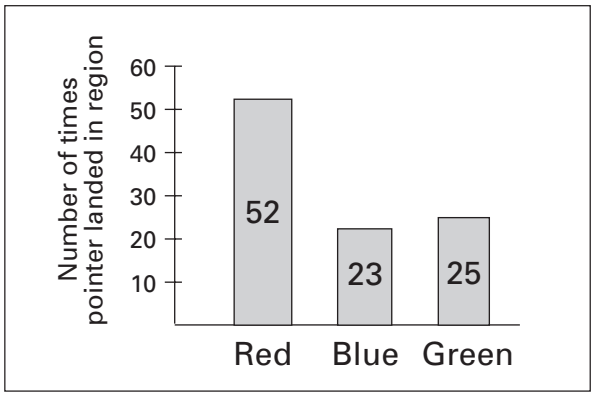


Spinner 2



Spinner 3

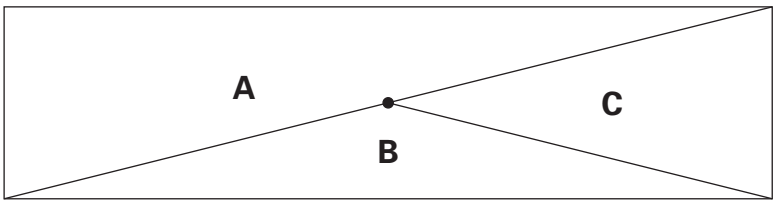
Data



On another sheet , explain how you decided to subdivide each spinner as you did.

Focus Student Activity 34.3

NAME _____ **DATE** _____



Spinner Z

1 Complete the following statement: "If Spinner Z were spun 100 times, we think that, theoretically the pointer would land in sector A about _____ times, in sector B about _____ times, and in sector C about _____ times." Here is how we decided these numbers:

2 Each of the 23 students in Ms. Miller's class spun Spinner Z 100 times. Following is the average number of times the pointer landed on each part of the spinner in 100 spins: A, 51 times; B, 32 times; and C, 17 times.

Ms. Miller wrote the following 3 predictions on the board:

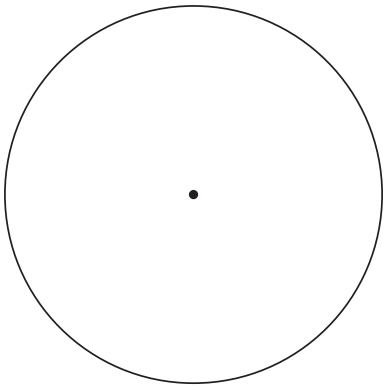
	A	B	C
Prediction 1	53	24	23
Prediction 2	51	41	8
Prediction 3	53	31	16

Before she spun Spinner Z another 100 times, Ms. Miller asked each student to vote for Prediction 1, 2, or 3. Students who voted for the prediction closest to her results would receive a prize. How would you vote and why?

Follow-up Student Activity 34.4

NAME _____ **DATE** _____

1 a) On the circle at the right, design a spinner with 5 sectors (A, B, C, D, and E) so that:



- $\frac{3}{8}$ of the area of the spinner is covered by sector A
- a central angle of 30° forms sector B
- sector C covers twice the area of sector B
- sector D covers $\frac{1}{3}$ as much as sector A
- sector E covers what is left

b) Record what you think should be the measure of the central angle that forms each sector on your spinner:

A_____ ; B_____ ; C_____ ; D_____ ; E_____

c) Tell how you decided the measures of the angles that form sectors A and E:

d) Tell what part of the circle is covered by each sector. Express each part as a fraction and as a decimal:

A_____ = _____; B_____ = _____; C_____ = _____;

D_____ = _____; E_____ = _____;

e) Explain how you determined the decimal value for A and B in d) above.

Continued on back.

Follow-up Student Activity (cont.)

2 Suppose the spinner you created in Problem 1 were spun 50 times. Draw and label 3 bar graphs so that:

a) Bar Graph I shows data that *would not be likely* to come from 50 spins of your spinner;

Bar Graph I

b) Bar Graph II shows data that would be *somewhat likely* to come from 50 spins of your spinner;

Bar Graph II

c) Bar Graph III shows data that would be *highly likely* to come from 50 spins of your spinner.

Bar Graph III

d) Explain how you decided on the data you used for Bar Graph III:

3 Conduct an experiment by spinning your spinner from Problem 1 50 times. On another sheet, make a bar graph, circle graph, and rectangular area model of your data. Explain how your data is like your predictions for Bar Graph III in 2c) above. How is it different and why do you think this is so?

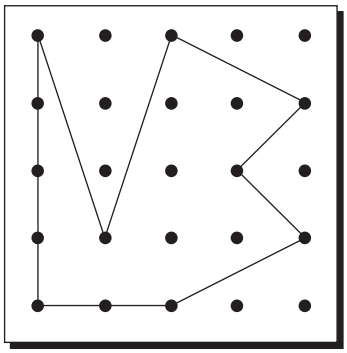
Follow-up Student Activity 35.1

NAME _____ **DATE** _____

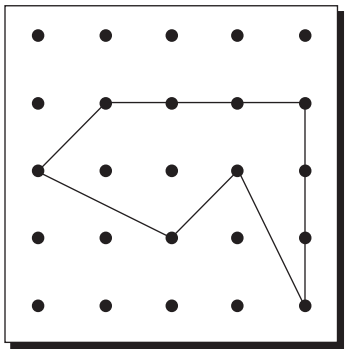
1 In your own words, explain the meaning of "area."

2 Determine the area of each geoboard region shown below. If needed, use the attached blank geoboard paper to explore your ideas. Identify the area unit(s) you use for each region.

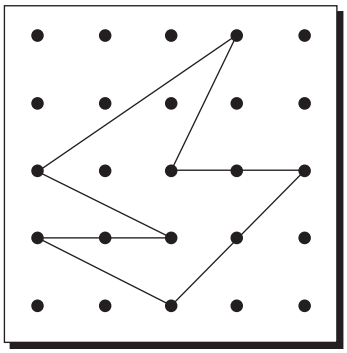
a)



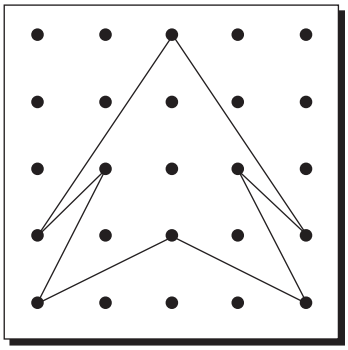
b)



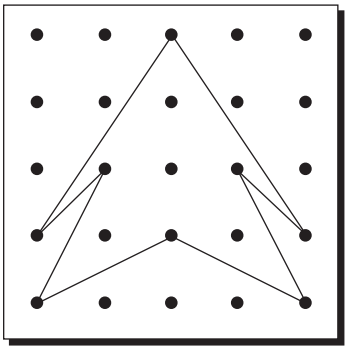
c)



3 Find the area of this region using at least 2 different methods. Explain each method.



Method 1:

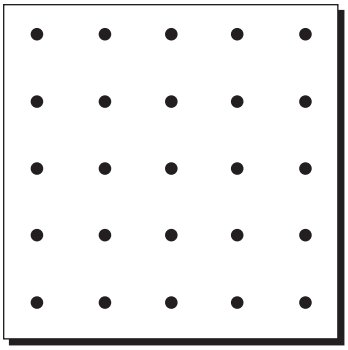


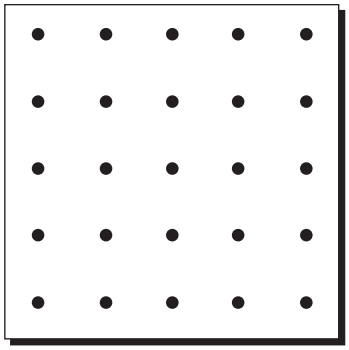
Method 2:

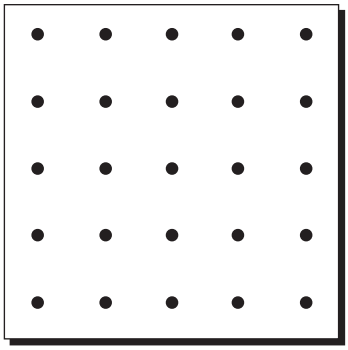
(Continued on back.)

Follow-up Student Activity (cont.)

4 On the blank geoboards below sketch 3 different regions whose areas you think would be challenging to find. Find and record the area of each of your figures. (Be sure to indicate what you use as the area unit.)

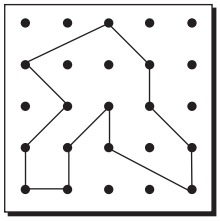


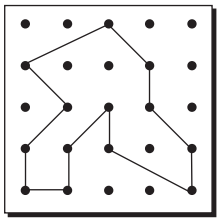


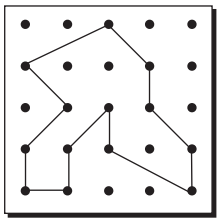


5 Determine the area of Shape X using each of area units A, B, and C.

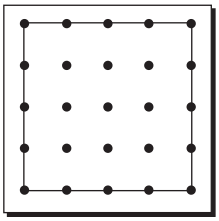
Shape X

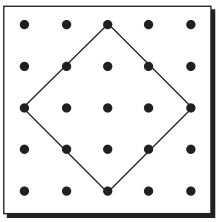


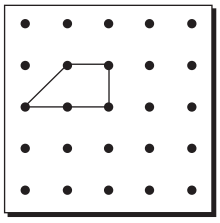




Unit

A 

B 

C 

Focus Master A

Follow-up Student Activity 36.1

NAME _____ **DATE** _____

1 On the attached base ten grid paper, draw a diagram of each of these situations. Next to each diagram, show the area and linear units you used.

a) A rectangle with dimensions 12 by 27.

b) A square whose area is 576.

c) A rectangle whose area is 391 and one dimension is 17.

d) A rectangle with area 693 and one dimension 21.

e) A square with area 1.96 square units.

f) A rectangle with area 1.14 square units and one dimension .3 linear units.

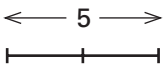
g) A rectangle with dimensions 2.3 linear units by 3.6 linear units.

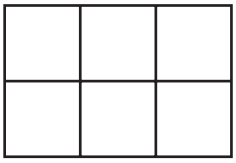
2 Determine the *dimensions*, *area*, and *perimeter* of each rectangle in Problem 1 by mentally counting the area and linear pieces you sketched. Record these measures on your drawings.

(Continued on back.)

Follow-up Student Activity (cont.)

3 Use the given lengths to help you determine the dimensions, area, and perimeter of each of the rectangles below:

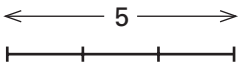
a) 

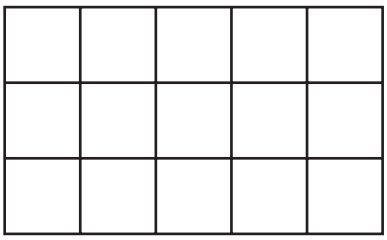


Dimensions: _____

Area: _____

Perimeter: _____

b) 



Dimensions: _____

Area: _____

Perimeter: _____

4 Explain the details of your methods of determining the area and perimeter of 3b).

5 Explain ways that you think dimensions, area, and perimeter are related and ways they are different.

Connector Student Activity 37.1

NAME _____ **DATE** _____

1 Your teacher has a Mystery Rectangle with the same area and dimensions as the 1.0 x 5.0 rectangle shown on the next page. However, your teacher's rectangle is covered (with no gaps or overlaps) by 21 noncongruent rectangular tiles. Each tile is bordered by grid lines and each tile is covered by a rectangular array of letters. Notice that Tile A has dimensions .4 linear unit by .2 linear unit, area .08 square unit, and it is covered by $4 \times 2 = 8$ A's. Below are a few clues about the Mystery Rectangle.

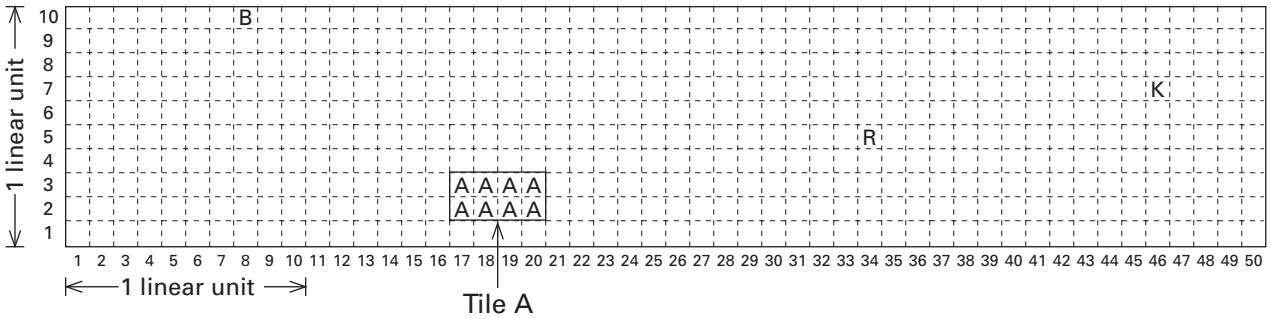
- Square (34, 5) contains an R, and Square (8, 10) contains a B.
- Tile B has area .56 square unit. It touches Tiles O, D, I, and J.
- The area of Tile C is double the area of Tile L.
- The area of Tile D is .24 square unit and its perimeter is 2.8 linear units.
- The area of Tile E is .24 square unit and one dimension is .8 linear unit.
- Tile F is not a square, but the number of F's that cover it is a square number.
- Tile G has area .45 square unit.
- The number of H's that cover Tile H is both prime and even.
- The area of Tile I is .22 square unit.
- Tile J has area .30 square unit.
- The area of Tile K is .63 square unit.
- The area of Tile L is .18 square unit.
- Tile M is a square.
- Tile N has area .15 square unit and perimeter 3.2 linear units.
- Tile O has area .36 square unit.

(Continued on back.)

Connector Student Activity 37.1 (cont.)

- The area of Tile P is .12 square unit.
- A prime number of Q's cover Tile Q.
- The number of R's that cover Tile R is neither a prime nor a composite number.
- The perimeter of Tile S is 1 linear unit and its area is .04 square unit.
- The perimeter of Tile T is 1.6 linear units and one dimension is .3 linear unit.
- Tile U is a square. Its area is .11 square unit more than the area of Tile M.

Mystery Rectangle



The class task (which may take several days) is to determine, using as few additional clues as possible, what each rectangular tile looks like and where it is placed on the Mystery Rectangle. On another sheet of paper (or in your journal), write other information and possibilities that you think are revealed by the above clues.

2 Tomorrow your group will complete a Clue Request Card with 3 questions like the ones shown below. What do you think your group should ask and why?

a) Is square _____ part of Rectangle _____? I think it could be because:

b) Is square _____ part of Rectangle _____? I think it could be because:

c) Is square _____ part of Rectangle _____? I think it could be because:

Focus Student Activity 37.2

NAME _____ **DATE** _____

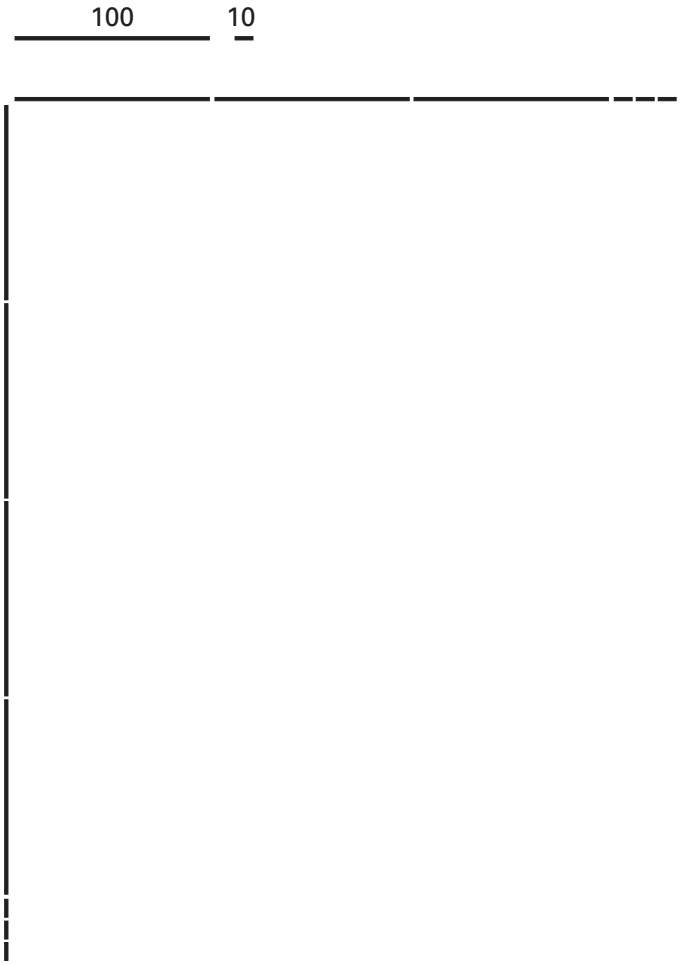
a) .1 .01

b) 10 1

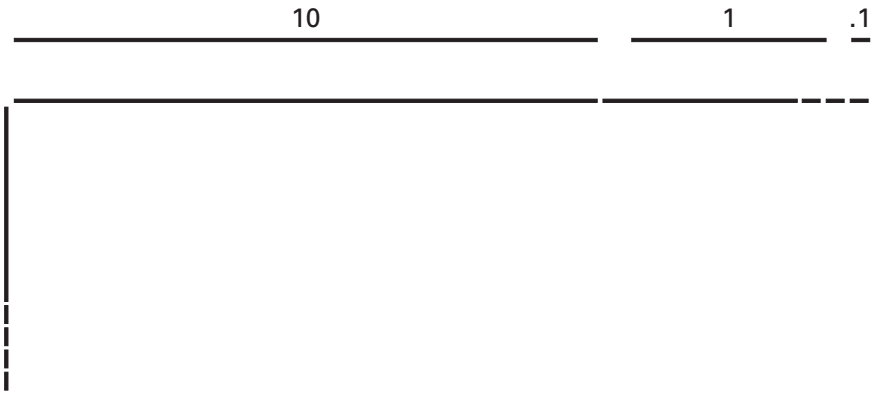
(Continued on back.)

Focus Student Activity 37.2 (cont.)

c)



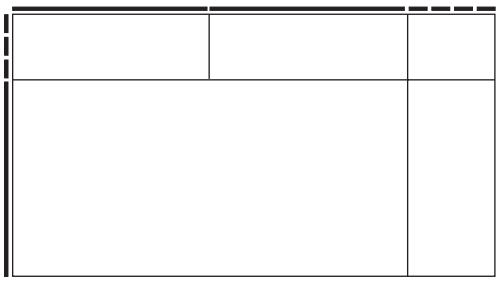
d)



Follow-up Student Activity 37.4

NAME _____ **DATE** _____

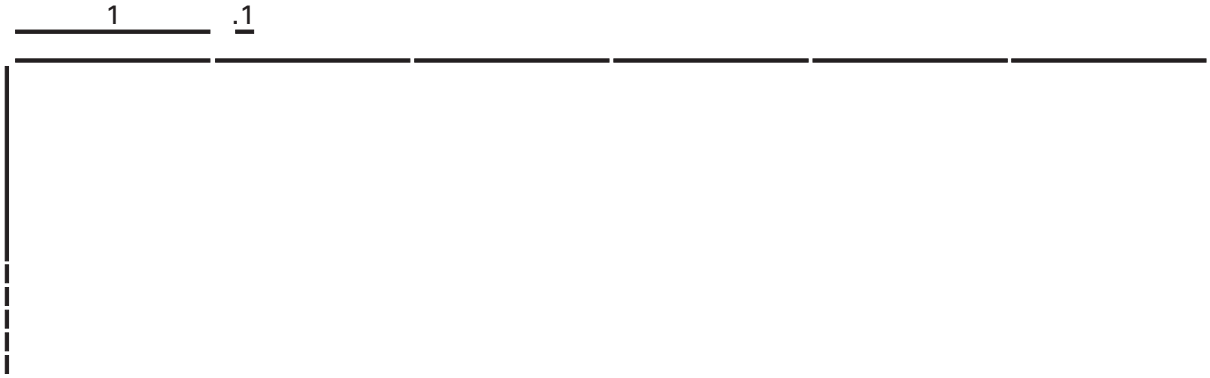
1 Simon said that he used the following diagram to compute 13 × 24. Write an equation that describes what you think might have been his thought processes.



2 Each arrangement of linear pieces below represents the dimensions of a rectangle. Imagine covering each rectangle with area pieces and mentally adding the areas of those pieces to find the area of the rectangle. Then sketch the rectangle to show how you counted the area and write equations to describe your thought processes.

a)

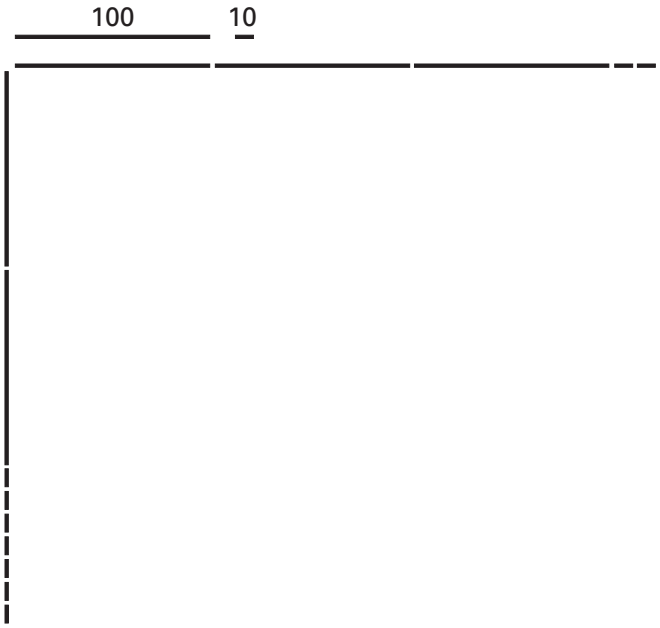
b)



(Continued on back.)

Follow-up Student Activity (cont.)

c)



d)

3 Determine and record the perimeter of rectangles a)-d) in Problem 2. Explain the method you used to find the perimeter of d).

4 Reggie plans to cover the floor of her room with carpet. The dimensions of the room are 3.6 meters by 4.2 meters. On another sheet of paper, write Reggie a letter that tells her how much carpet to buy. Provide a convincing explanation of why that is the right amount. Include diagrams to support your explanation.

5 Each of the following is the answer to a multiplication problem. For each answer in a)-d), on the attached grid paper write an interesting word problem that involves multiplying to find that answer. Show diagrams that illustrate how to find the answer to each of your problems.

a) .48 b) 38000 c) 5.4 d) 10.8

Follow-up Student Activity 38.1

NAME _____ **DATE** _____

1 Use area and linear pieces to find the quotients in these problems. Then sketch a diagram of your methods and solutions on the attached base ten recording paper. Next to each diagram, write an explanation of your reasoning and methods.

a) 4.8 ÷ 1.2 b) 344 ÷ 16 c) 2.73 ÷ 1.2

2 Use a calculator to find 57904 ÷ 112 = _____.

a) Tell which keys you pressed, in the order that you pressed them:

b) Do you think your calculator quotient is reasonable? Describe the strategies you used to decide about its reasonableness.

c) Write an interesting word problem for which you need to compute 57904 ÷ 112 to solve the problem.

(Continued on back.)

Follow-up Student Activity (cont.)

Dear _____,

Part of my math assignment today is to spend 20-30 minutes presenting to an adult my understanding of multiplication and division of decimals and whole numbers. Would you please take the time to listen to my presentation? The purpose of explaining to you is so I can clarify my thinking about these concepts and identify places I still have questions, so we can explore those questions more in class.

I will use the base ten pieces we use in class to show you how I view the meanings of multiplication and division and to show you ideas and methods that I think are especially interesting, easy, and/or challenging. If I get stuck or have difficulty explaining an idea, would you remind me to write it down so that I can take my questions back to class and investigate them some more?

When I am finished, please write a note to my teacher, listing three positive comments about my presentation. Thanks for taking the time to listen to my thinking!

Student Signature _____

_____ spent _____ minutes helping me understand her/his thinking about multiplication and division of decimals. Following are three positive comments about that presentation.

1.

2.

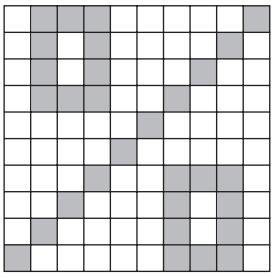
3.

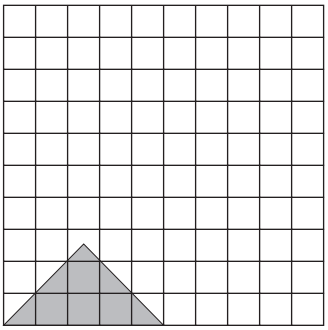
Other positive thoughts:

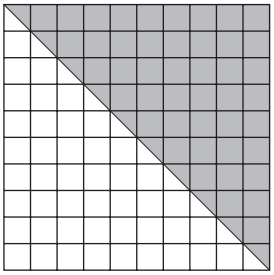
Adult Signature _____ Date _____

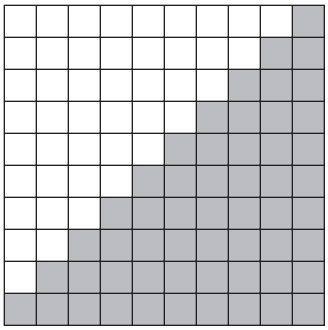
Focus Student Activity 39.1

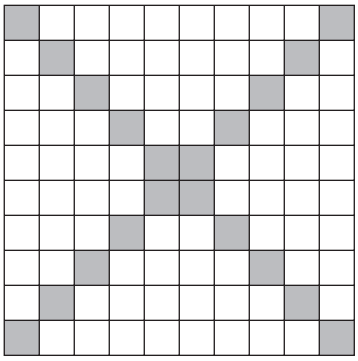
NAME _____ **DATE** _____

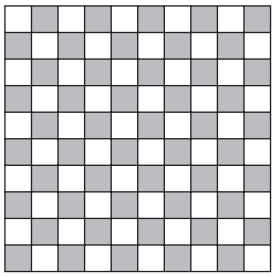
a)


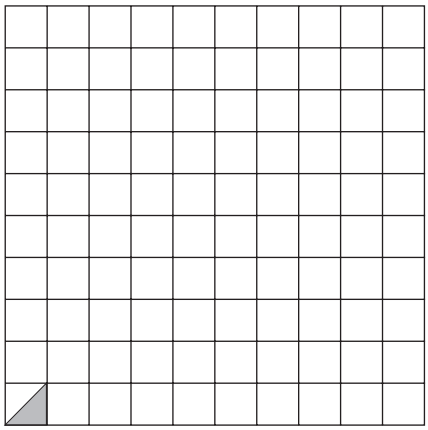
b)


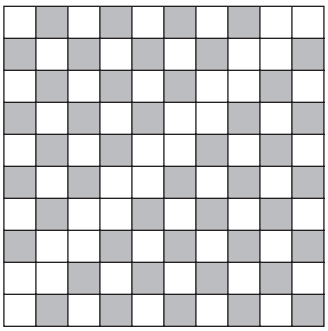
c)


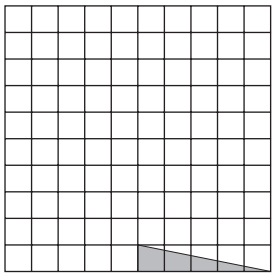
d)


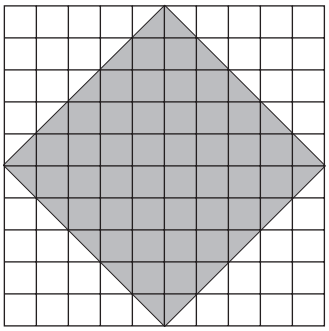
e)


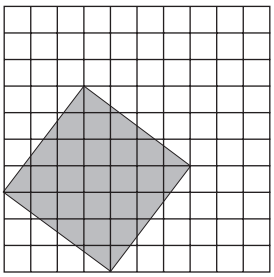
f)


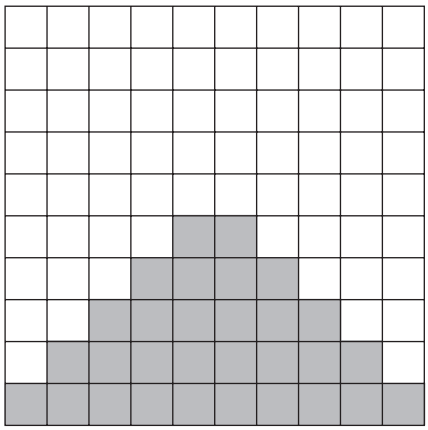
g)


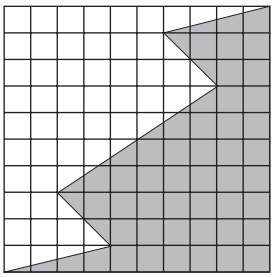
h)


i)


j)


k)


l)


m)


Focus Student Activity 39.2

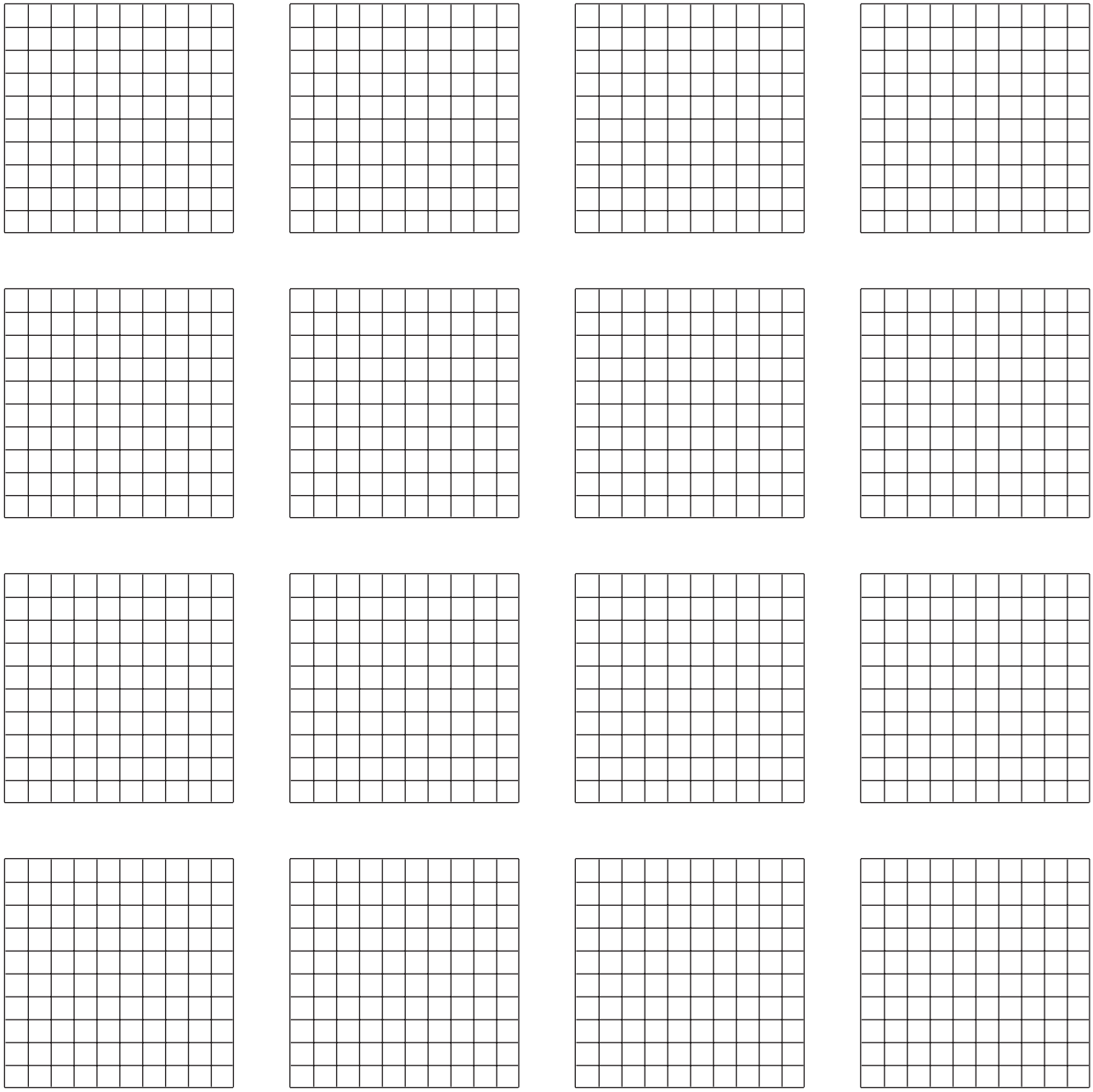
NAME _____ **DATE** _____

Using the grids below, shade each of the following amounts:

a) 25% b) $12\frac{1}{2}$% c) $\frac{7}{2}$% d) .75% e) $\frac{1}{4}$%

f) 99.9% g) 5.5% h) 137% i) 210%

Label the grids.



Follow-up Student Activity 39.3

NAME _____ **DATE** _____

1 Write an explanation, in your own words, of the meaning of percent.

2 Use your base ten pieces to build a model of the relationships in each of the following situations. Then sketch your model on the attached sheet of percent grids. Next to each diagram, write at least two different mathematical observations that you can make by looking at your model. (Be sure to label each diagram.)

a) 30% of the football players injured a knee during the football season. 15 players had knee injuries this season.

b) A television that usually sells for $210 is on sale for 15% off the regular price.

c) Last year 230 students joined the choir. This year 253 students joined.

d) The drama students need to raise $2000 for a class trip. They have raised $880 so far.

e) Dylan has earned $115 mowing lawns. This is 46% of what he needs to buy the bike he wants.

f) In an experiment, Willis' spinner landed on purple 32% of the time. He spun the spinner a total 75 times.

g) This year tickets to the concert cost $15, which is 125% of the cost last year.

(Continued on back.)

Follow-up Student Activity (cont.)

3 Spend 20 or more minutes teaching an adult about the meaning of percent. Use your base ten pieces to give examples and illustrate your thinking. Then pick three situations from Problem 2 and explain to the adult:

a) your model for each situation,

b) your mathematical observations about each model.

Be sure they understand your thinking. In this space have the adult write 3 "I appreciate…" and 1 "I wish…" statement about your presentation.

4 Find at least three different advertisements or articles in the newspaper or a magazine that involve percent. Attach them to this assignment.

Follow-up Student Activity 40.1

NAME _____ **DATE** _____

For this activity, you need to get a set of segment strips from your teacher.

1 Suppose 2 pink segments have total length 1 linear unit. Draw a diagram to show how to use this information to determine the length of 3 blue segments.

2 Suppose 4 green segments have a total length of 7 linear units. Draw a diagram and explain how to use this information to determine a length of $7/4$.

3 Suppose the length of 1 pink segment is $5/4$ linear units. Show and explain how to use this information to draw a length of 5 linear units.

(Continued on back.)

Follow-up Student Activity (cont.)

4 For each of the following conditions, draw and label the length described (make sure your sketch shows how your length satisfies the conditions).

a) Suppose the length of 1 white segment is 1 linear unit. Mystery Length I is formed by $3\frac{2}{3}$ blue segments placed together end-to-end. How long is Mystery Length I?

b) Suppose 3 blue segments have total length $2\frac{1}{4}$ linear units. Draw 1 linear unit.

c) Suppose the length of 1 white segment is 1 linear unit. $2\frac{1}{2}$ copies of Mystery Length II placed end-to-end measure a total of $1\frac{1}{4}$ linear units. How long is Mystery Length II?

d) Suppose 4 yellow segments have length 3 linear units and Mystery Length III is $3 \div 4$ linear units. Draw Mystery Length III.

e) Suppose the length of 1 green segment is 1 linear unit. The total length of 6 pink segments is $\frac{3}{4}$ as long as Mystery Length IV. How long is Mystery Length IV?

5 On another sheet, write a letter to a student who wasn't in class for the explorations with the segment strips. Send them a set of colored segment strips and explain how you think they can be used to understand the meaning of equivalent fractions and ways to determine them.

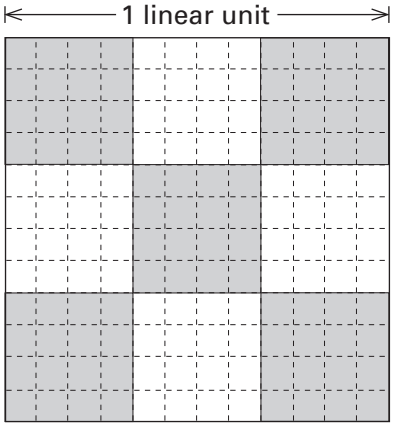
Focus Student Activity 41.1

NAME _____ **DATE** _____

For each of the following rectangles, determine:

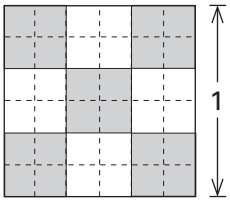
1. the total *area* of the shaded part of the rectangle;
2. what *fraction* of the rectangle's area is shaded;
3. the *perimeter* of the rectangle.

a)



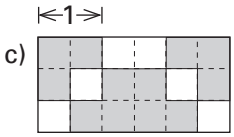
1. _____
2. _____
3. _____

b)



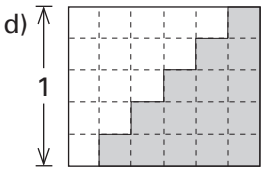
1. _____
2. _____
3. _____

c)



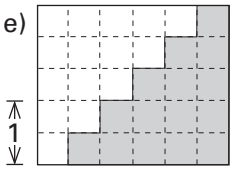
1. _____
2. _____
3. _____

d)



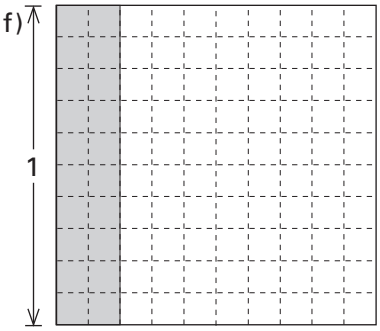
1. _____
2. _____
3. _____

e)



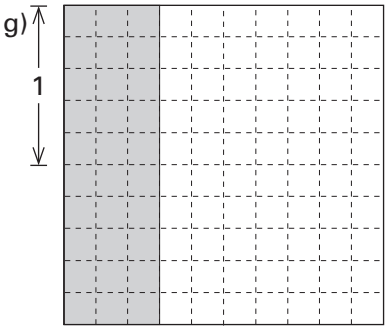
1. _____
2. _____
3. _____

f)



1. _____
2. _____
3. _____

g)

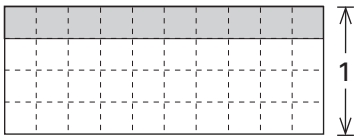


1. _____
2. _____
3. _____

(Continued on back.)

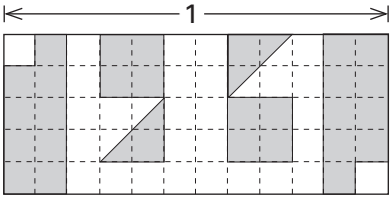
Focus Student Activity 41.1 (cont.)

h)



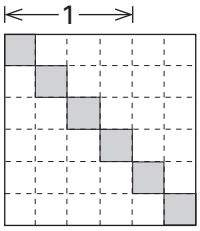
1. _____
2. _____
3. _____

i)



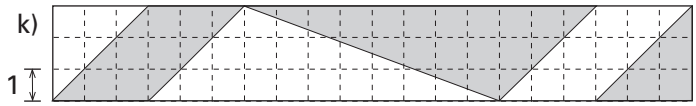
1. _____
2. _____
3. _____

j)



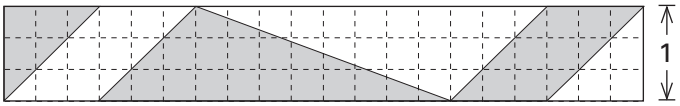
1. _____
2. _____
3. _____

k)



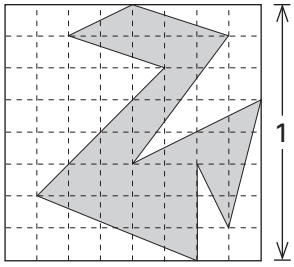
1. _____
2. _____
3. _____

l)



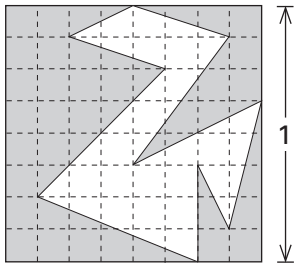
1. _____
2. _____
3. _____

m)



1. _____
2. _____
3. _____

n)



1. _____
2. _____
3. _____

Follow-up Student Activity 41.2

NAME _____ **DATE** _____

1 Let the length of 1 green segment be 1 linear unit. Draw a unit square on the attached grid paper. Then do the following:

Color $\frac{1}{3}$ of the square blue.
Color $\frac{1}{2}$ of the uncolored part red.
Color $\frac{1}{6}$ of what is now uncolored green.
Color $\frac{3}{5}$ of what is now uncolored orange.
Color $\frac{1}{4}$ of what is left purple.
Cut out your colored square and tape it at the right.

Now tell what part of the unit square is not colored: _____

Tell the area of the region covered by each color.

Blue: _____ Orange: _____

Red: _____ Purple: _____

Green: _____

2 Suppose the linear unit is the length of 2 yellow segments. Using this linear unit, what is the area of each differently-colored region on your drawing in Problem 1? Explain or draw a diagram to show how you decided these areas.

Blue: _____

Red: _____

Green: _____

Orange: _____

Purple: _____

(Continued on back.)

Follow-up Student Activity (cont.)

3 Suppose the length of 1 yellow segment is $\frac{2}{3}$ linear unit. Write at least 6 mathematical observations that involve fractions and are about relationships between Figure A and Figure B below.

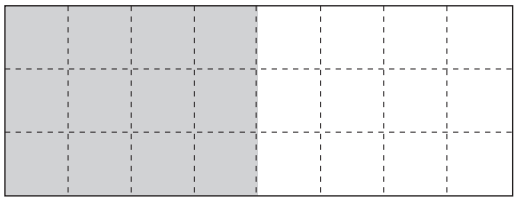


Figure A

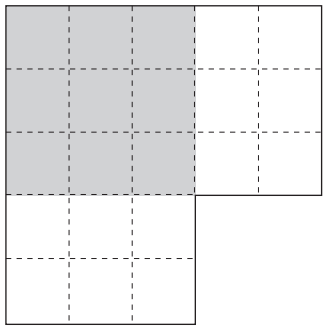


Figure B

4 Suppose the length of 3 yellow segments is 1 linear unit. On the attached grid paper draw a polygon with area 1 square unit. Subdivide your polygon into 4 colored regions (red, blue, green, and yellow) so that:

- There are no gaps or overlaps of colors.
- $\frac{1}{6}$ of the polygon is red.
- $\frac{3}{8}$ of the polygon is blue.
- $\frac{3}{11}$ of what is neither red nor blue is green.
- There is $2\frac{2}{3}$ as much yellow as there is green.

Next to your polygon, please do the following:

a) tell what part of the polygon is covered by the red and green regions combined;

b) tell how many red regions it would take to exactly cover a blue region and explain how you determined this;

c) determine the difference between the area of the yellow region and the area of the red region and explain your methods;

d) write three more interesting questions involving fractions about your colored polygon and then answer your questions.

Follow-up Student Activity 42.2

NAME _____ **DATE** _____

1 On another sheet of paper, tape or glue a model of each of the following situations. Then, next to each model write at least 3 observations regarding mathematical relationships you can "see" in the model. Whenever possible write number statements or equations to represent your observations.

a) $2\frac{1}{8}$ copies of a length of $\frac{1}{2}$ linear unit are joined together end-to-end.

b) This rectangle has dimensions $\frac{5}{6}$ linear unit by $\frac{4}{5}$ linear unit.

c) These 2 rectangles both have area $1\frac{1}{8}$ square units, but their perimeters are different.

d) If $3\frac{1}{3}$ copies of a certain Mystery Length are placed end-to-end, their total length is 5 linear units.

2 Use your segment strips only to find the answer to each of the following. On another sheet, draw or tape the strips to illustrate your methods. Label each diagram and add a few comments to explain each step of your methods with the strips.

a) $\frac{3}{8} + \frac{1}{3}$ b) $\frac{4}{5} - \frac{1}{4}$ c) $\frac{3}{4} \times \frac{5}{7}$ d) $\frac{5}{6} \div \frac{3}{4}$

3 Repeat Problem 2 but this time using area representations of each problem. Cut out your models from $\frac{10}{12}$-cm grid paper and tape them in place on another sheet. Label your models and add comments to explain each step of your thought processes.

4 On the attached grid paper, write 4 fraction computations, 2 easy and 2 hard. (Don't use the problems from 2 and don't solve the problems yet!).

a) Label the 2 computations you think are easy and the 2 that are hard. Tell why you think each problem is easy or hard.

(Continued on back.)

Follow-up Student Activity (cont.)

b) Now show how to solve the 4 problems using segment strips or area representations on grid paper.

5 Suppose the length of 1 white segment is 1 linear unit. Cut out 3 noncongruent rectangles, each with area 1 square unit. Arrange these rectangles in order by size of their perimeters and tape them onto another sheet. Record the dimensions, area, and perimeter of each rectangle.

6 Suppose the length of 1 white segment is 1 linear unit. Cut out 3 noncongruent rectangles, each with perimeter 1 linear unit. Arrange these rectangles in order by the size of their areas and tape them onto another sheet. Record the dimensions, area, and perimeter of each rectangle.

7 Record at least 1 multiplication and at least 1 division equation represented by each rectangle in Problems 5 and 6. (Write these equations next to the rectangles.)

8 Write at least 3 observations and/or conjectures based on mathematical relationships you notice in the rectangles that you formed in Problems 5 and 6.

Follow-up Student Activity 43.1

NAME _____ **DATE** _____

For this activity you need a set of segment strips, 2 sheets of $^{10}/_{12}$-cm grid paper, and 1 sheet of 1-cm grid paper.

1 Use the distance between 2 "tall" marks on the red segment strip as the linear unit. Find the length as accurately as possible of each of the following. Write each length as a fraction, a decimal, and a percent of 1 unit.

a) 1 pink c) 3 green e) 1 blue

b) 5 orange d) 3 white f) 5 yellow

g) Explain how you determined the fraction, decimal, and percent for f).

2 Find a close approximation of the distance between 2 "tall" marks on the red segment strip when each of the following is the linear unit.

a) 1 pink c) 3 green e) 1 blue

b) 5 orange d) 3 white f) 5 yellow

3 Using the length of 1 white segment as the linear unit, do the following on a sheet of 1-cm grid paper:

a) draw a rectangle whose dimensions are $1\frac{3}{5}$ linear units by $2\frac{1}{2}$ linear units;

b) explain how to use your diagram to find the area and perimeter of the rectangle;

c) write the area and perimeter as a fraction and as a decimal.

(Continued on back.)

Follow-up Student Activity (cont.)

4 Now shade 10% of your rectangle in Problem 3.

a) Next to the diagram tell how you determined what to shade.

b) Tell the total area of the shaded part and explain how you decided this.

5 Use segment strips to teach an adult why $\frac{3}{5} + \frac{1}{3} = \frac{14}{15}$. On another sheet of paper tell who you "taught" and describe (include diagrams) each step of your demonstration.

6 Model each of the following situations with segment strips and/or grids. On a separate sheet, draw a picture of each of your models (or tape your models to the sheet). Next to each model write at least three interesting mathematical questions *whose answers could be found by studying your model*. (Write the answers to your questions, too!)

a) The music teacher roped off an area in front of the stage that was equal to $\frac{1}{3}$ the area of the stage. The dimensions of the stage are 8 yards by 4 yards.

b) To get in shape for a track meet Travis ran the following distances last week: Monday, $1\frac{1}{2}$ miles; Tuesday, 2 miles; Wednesday, $1\frac{2}{5}$ miles; Thursday, 1.9 miles; and Friday, $2\frac{3}{10}$ miles.

c) Josh weighs 3.5 times as much as his little brother. Josh weighs 84.7 pounds.

d) In the 3-mile Spring Fun Run, Erin had run 80% of the 3 miles when she stopped for a drink of water.

7 Pick one or more of the situations in Problem 6 to model (with segment strips or grids) for an adult. Explain to them how you were able to answer your questions by using the model. On another sheet, tell who you taught, which situation(s) you modeled, and the adult's reactions.

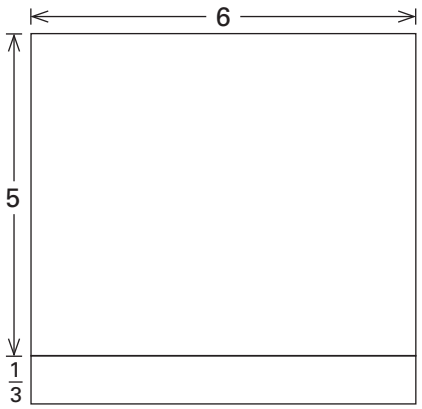
Connector Master A

Connector Student Activity 44.1

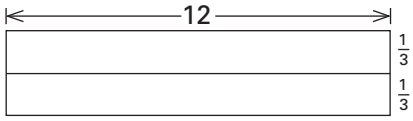
NAME _____ **DATE** _____

Here are diagrams some students sketched to solve these problems. Next to each diagram explain how you think each student thought about the problem.

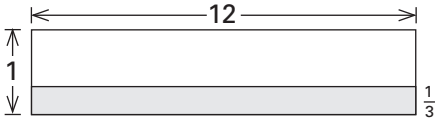
a) $5\frac{1}{3} \times 6$



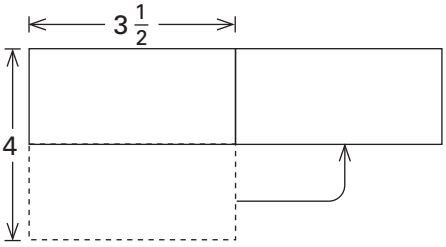
b) $\frac{2}{3} \times 12$



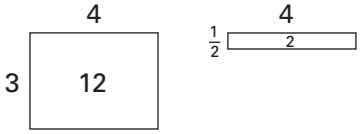
c) $\frac{2}{3} \times 12$



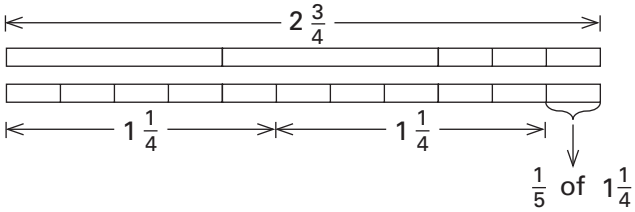
d) $4 \times 3\frac{1}{2}$



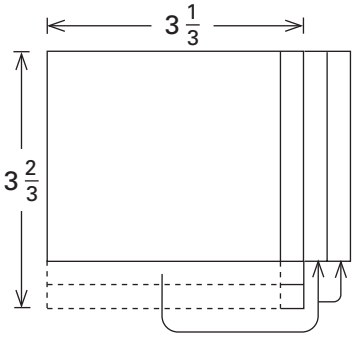
e) $3\frac{1}{2} \times 4$



f) $2\frac{3}{4} \div 1\frac{1}{4}$



g) $3\frac{2}{3} \times 3\frac{1}{3}$



Focus Master B

Use segment strips and/or $^{10}/_{12}$-cm grid paper to find the sum, difference, product, *and* quotient of the numbers on your card. Then in order to clearly communicate the "story" of your procedures with the strips and/or grids, create a poster with the following information.

1. For each computation that you did, tell the story of your actions using each of the following:

   a) a *pictorial representation* of your methods (a diagram or model that illustrates your methods—no words);

   b) a *verbal representation* of your methods (a description in words only);

Try to make each representation above so clear that your reader could duplicate your actions with the strips and/or grids simply by studying that representation. (You could test your ideas by having someone in another group study a representation and try to duplicate your actions with no other clues.)

2. For the sum and difference you computed invent a symbolic representation of your methods using numbers and arithmetic symbols only—no words or pictures.

Other Ideas

Here are some ways you could go "above and beyond" on your poster:

• explain and illustrate what happens if you reverse the order of the numbers in each of your 4 computations;

• determine whether any of your methods generalize (that is, would they work for *any* pair of fractions or mixed numbers?);

• make a symbolic (numbers and math symbols only) representation of your methods of using strips or grids to find the product and/or quotient of your numbers;

• add diagrams that show ways to solve each computation based on *all* the different meanings of each operation.

Focus Master C

Each set of symbols below tells the "story" of a student's actions with segment strips or grids (or sketches of them) to solve a problem. Draw or cut out diagrams to show what you think were each student's actions with strips or grids and write a brief explanation of each diagram. Will each method work on other problems?

Keisha, $3^3/_5 - 2^2/_3$

$$\begin{array}{lcl} 3^3/_5 & = & 3^9/_{15} \\ \underline{2^2/_3} & = & \underline{2^{10}/_{15}} \\ & & \underline{\underline{1^4/_{15}}} \end{array} \quad \begin{array}{c} = \\ = \end{array} \quad \begin{array}{r} 2^{24}/_{15} \\ \underline{2^{10}/_{15}} \\ \end{array}$$

John, $4^1/_5 - 1^3/_5$

$$\begin{array}{r} 4^1/_5 \\ -\ \underline{1^3/_5} \end{array} \rightarrow \begin{array}{r} 4^3/_5 \\ -\ \underline{2} \\ \underline{\underline{2^3/_5}} \end{array}$$

Lisa, $3^3/_5 + 2^2/_3$

$$\begin{array}{lcl} 3^3/_5 & = & {}^{18}/_5 \\ \underline{2^2/_3} & = & \underline{{}^8/_3} \\ & & \underline{\underline{{}^{94}/_{15}}} \end{array} \quad \begin{array}{c} = \\ = \end{array} \quad \begin{array}{r} {}^{54}/_{15} \\ \underline{{}^{40}/_{15}} \\ \end{array}$$

Colleen, $2^5/_7 + 1^1/_2$

$$\left.\begin{array}{l} 2^5/_7 \\ 1^1/_2 \end{array}\right\} \quad \left.\begin{array}{l} 2^{10}/_{14} + {}^4/_{14} \\ 1^7/_{14} - {}^4/_{14} \end{array}\right\} \quad \begin{array}{r} 3 \\ \underline{1^3/_{14}} \\ \underline{\underline{4^3/_{14}}} \end{array}$$

David, $3^3/_5 \times 5^5/_6$

$(3 \times 5) + (3 \times {}^5/_6) + ({}^3/_5 \times 5) + ({}^3/_5 \times {}^5/_6)$
$= 15 + {}^{15}/_6 + 3 + {}^3/_6$
$= 18 + {}^{18}/_6$
$= 18 + 3 = \underline{\underline{21}}$

Juanita, $4^1/_2 - 3^1/_7$

$3^1/_7 + {}^6/_7 = 4$
$4 + {}^1/_2 = 4^1/_2$
${}^6/_7 + {}^1/_2 = {}^{12}/_{14} + {}^7/_{14} = \underline{\underline{{}^{19}/_{14}}}$

Daniel, $4^2/_5 \times 1^1/_2$

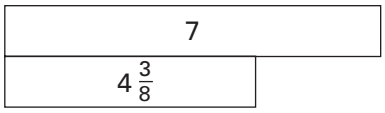
$2^1/_5 \times 3$
$= (2 \times 3) + ({}^1/_5 \times 3)$
$= 6 + {}^3/_5$
$= \underline{\underline{6^3/_5}}$

Follow-up Student Activity 44.2

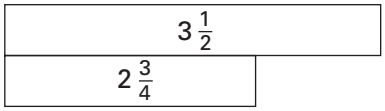
NAME _____ **DATE** _____

1 On another sheet of paper, show how to use segment strips and/or $^{10}/_{12}$-cm grids to find the sum, difference, product, *and* quotient of $2\frac{1}{2}$ and $1\frac{3}{4}$. You may either make a diagram of your segments or attach segments to your paper. Next to each diagram, write number statements or equations and as few words as possible to describe each step of your methods.

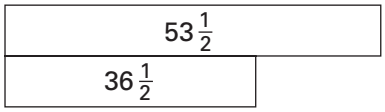
2 What are 3 pairs of numbers whose difference is the same as $7 - 4\frac{3}{8}$? Tell how you determined these numbers. Circle the pair whose difference you prefer to compute.



3 What must you add to $2\frac{3}{4}$ to equal $3\frac{1}{2}$? Explain how you decided this.



4 What are 3 pairs of numbers whose sum is the same as $36\frac{1}{2} + 53\frac{1}{2}$. Tell how you determined these numbers. Circle the pair whose sum you prefer to compute.



(Continued on back.)

Follow-up Student Activity (cont.)

5 For each set of numbers and operation(s) listed below, describe an everyday situation that involves the numbers and operation(s).

a) $\frac{3}{4}$, $\frac{2}{3}$; multiplication

b) $2\frac{1}{2}$, $3\frac{5}{6}$, $1\frac{3}{4}$; addition

c) $7\frac{4}{5}$, $2\frac{1}{3}$; subtraction

d) $7\frac{1}{2}$, $1\frac{1}{3}$; division

e) $1\frac{1}{4}$, $2\frac{1}{3}$; multiplication and division

f) $2\frac{1}{8}$, $3\frac{1}{4}$, $1\frac{1}{6}$, and 4; addition, subtraction, multiplication, and division

Follow-up Student Activity 45.1

NAME _____ **DATE** _____

1 There are 30 tile in a paper sack. The tile have the colors red, yellow, blue, and green. To predict the contents of the sack, 4 students each conduct an experiment. The procedure they use is: without looking, draw one tile; record its color; return the tile to the sack; and then repeat for a total of 30 draws each. The numbers of each color these students obtained is shown below.

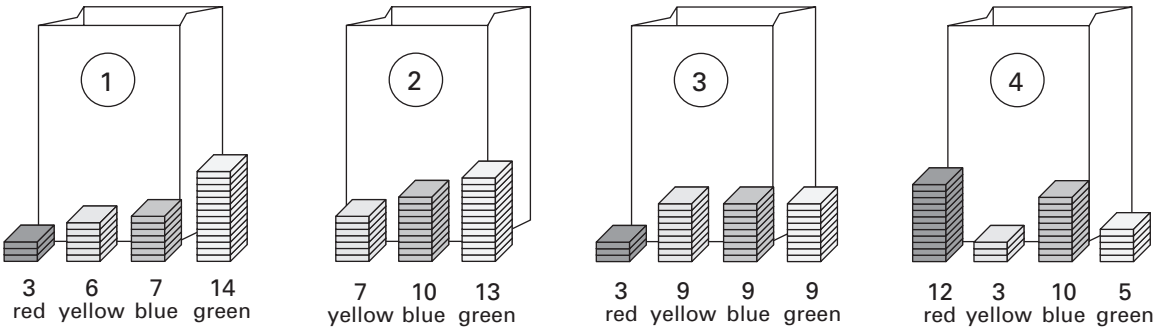
	Janine	Eric	Tyson	Rachelle
Red	1	2	0	4
Yellow	8	5	8	3
Blue	7	10	9	5
Green	14	13	13	18

Based on the information above, write what you think is likely to be the contents of the sack. Give valid mathematical arguments, supported by clear diagrams, to support your prediction.

(Continued on back.)

Follow-up Student Activity (cont.)

2 Compare the following sacks to what you think might be in the original sack in Problem 1. Label each sack as either Impossible, Unlikely, Not Sure, or Likely, and write a reason for each of your decisions.



3 Explain what you believe are the most important points to consider when you need to use sampling to make a confident prediction.

4 On another sheet of paper, make a diagram to show the relationship described in each of these situations. Next to each diagram write 3 or more interesting mathematical observations or predictions about the situation.

a) A radio station placed contest entry forms in a box with the names of 50 people of age 9-12, 70 people of age 13-19, and 80 people of age 20 or older. The winner will be selected by a random draw from the box.

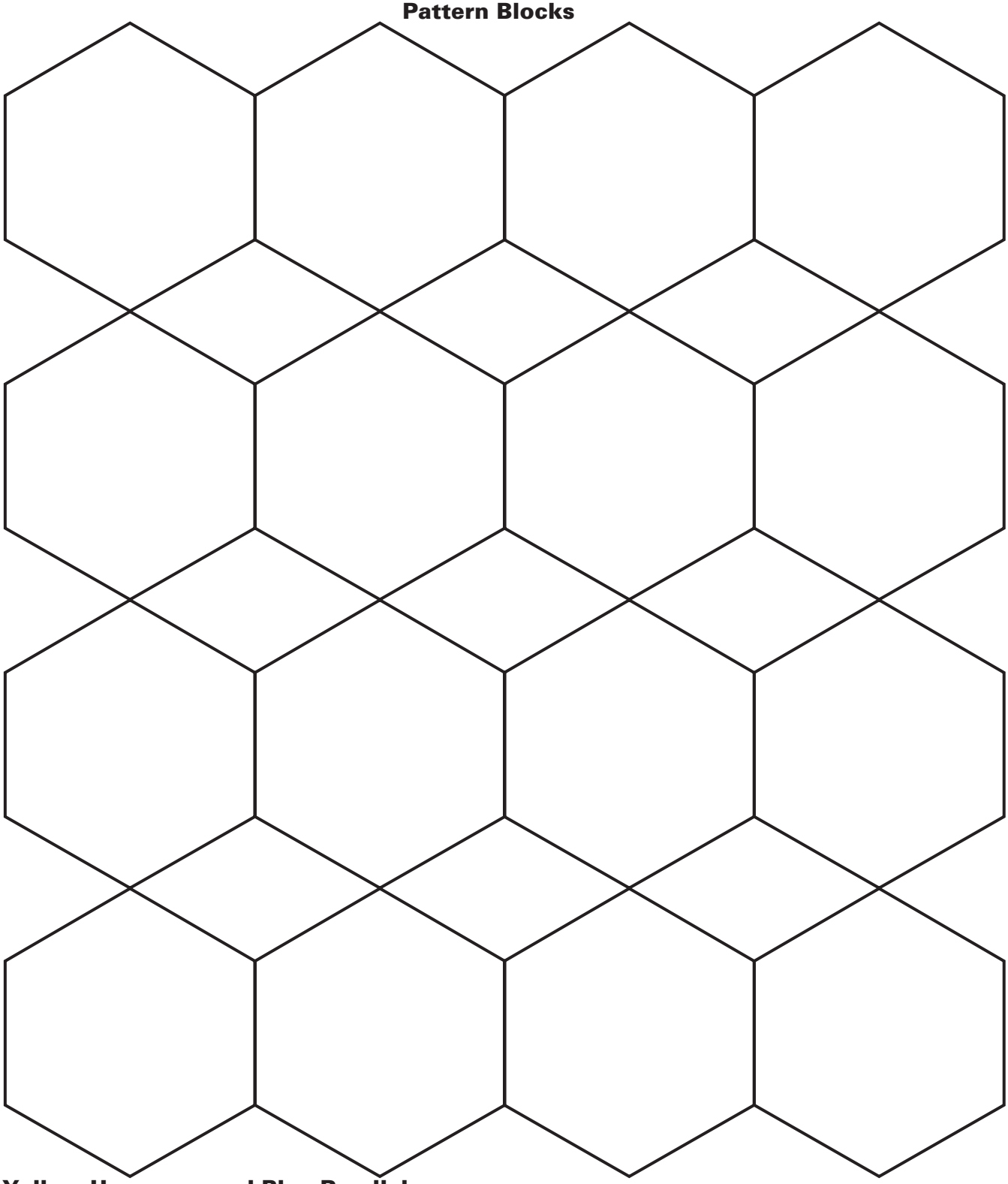
b) There are 60 pieces of bubble gum with colors of blue, yellow, and green in a machine. The theoretical probability of selecting a blue is $3/10$, and the probability of selecting a green is 50%.

visual mathematics

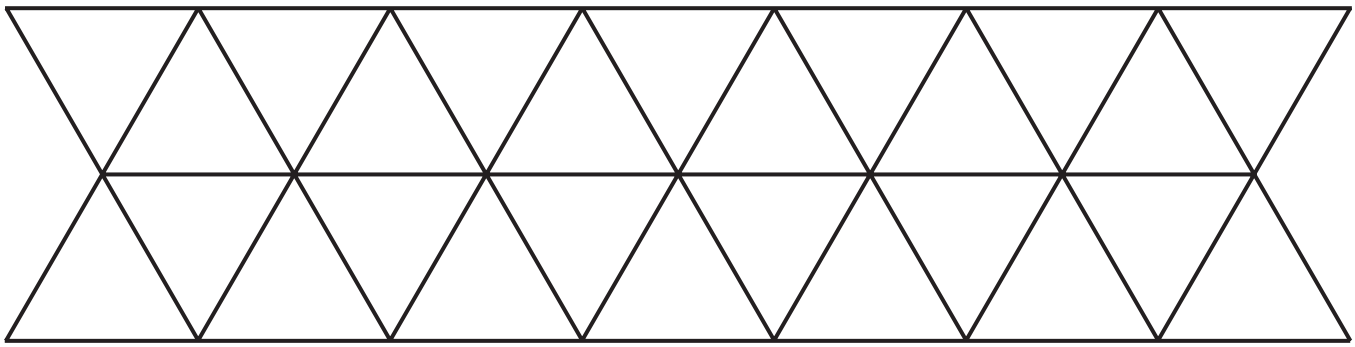
COURSE I STUDENT ACTIVITIES

Tools

Pattern Blocks

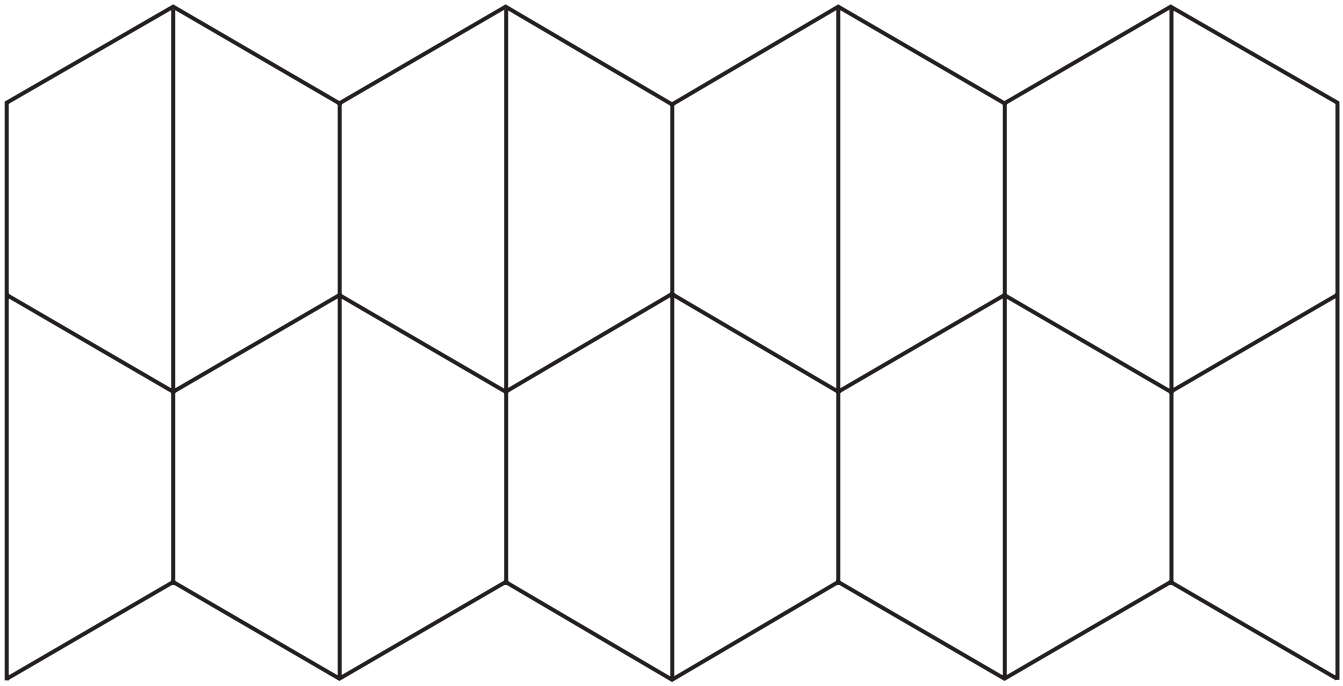


Yellow Hexagons and Blue Parallelograms

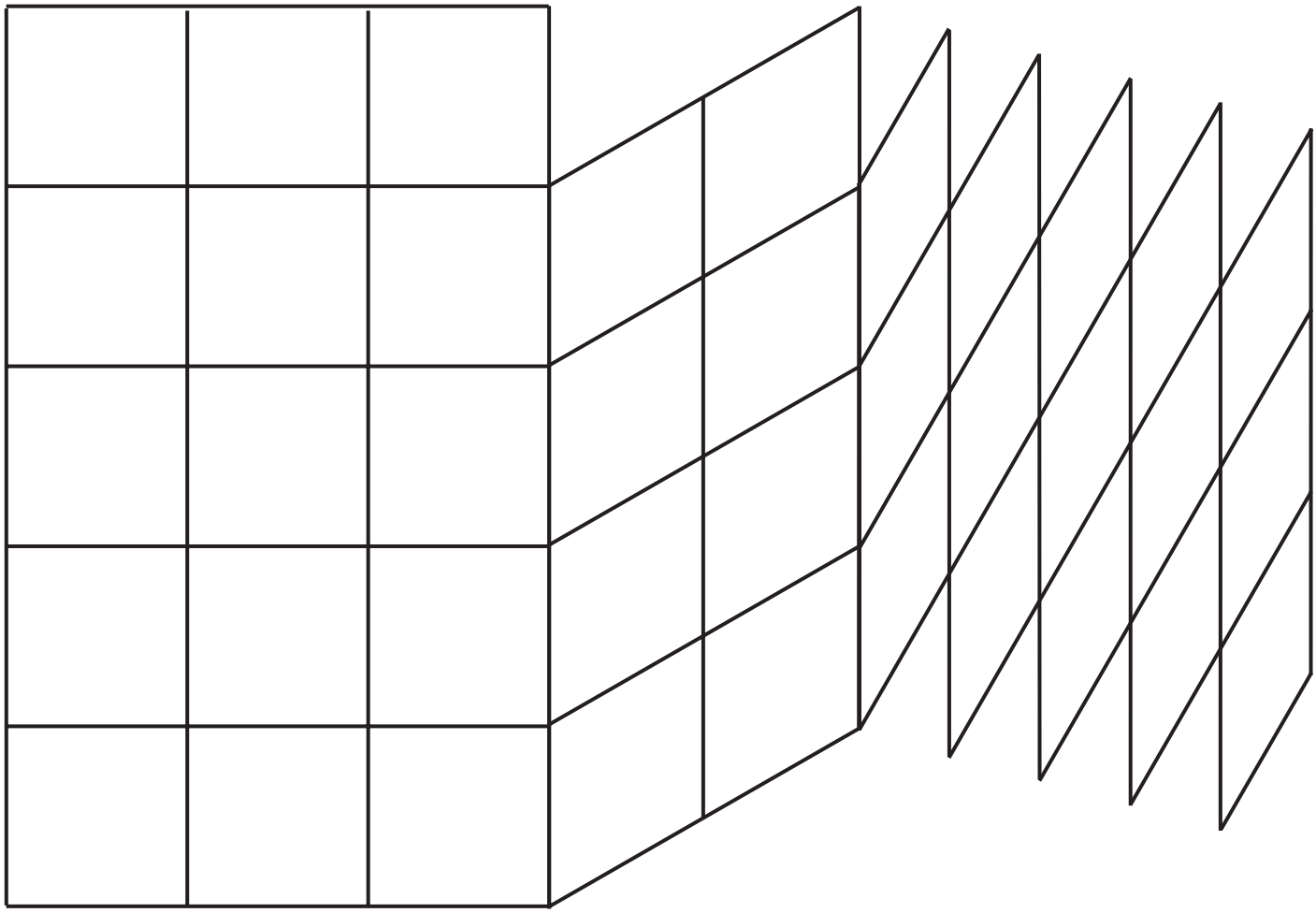


Green Triangles

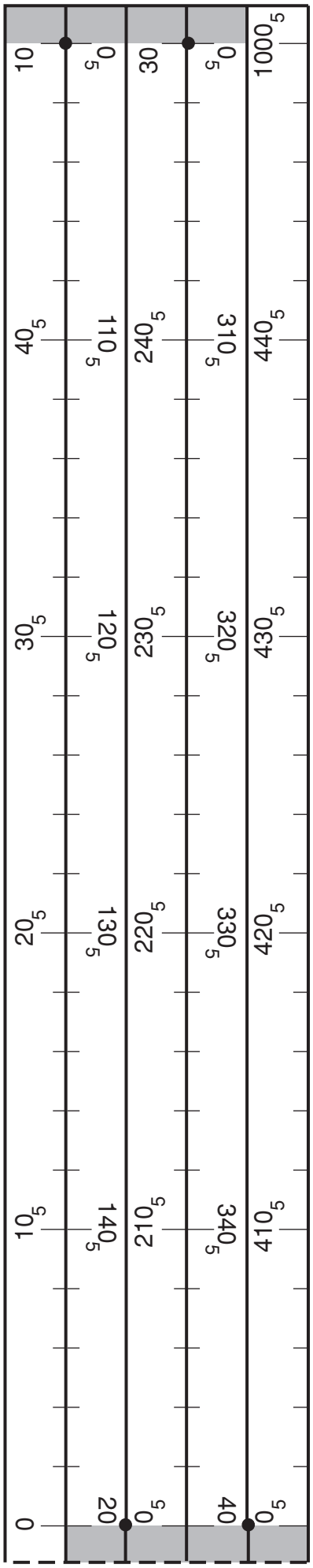
Pattern Blocks



Red Trapezoids

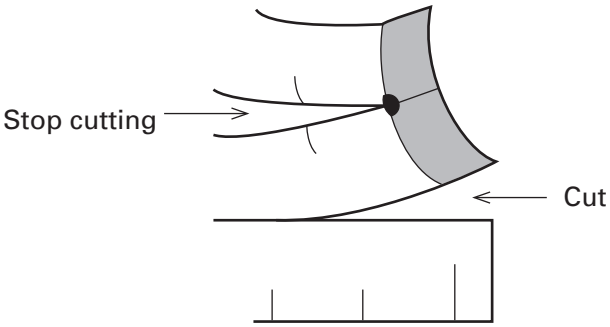


Orange Squares **Blue Parallelograms** **White Parallelograms**

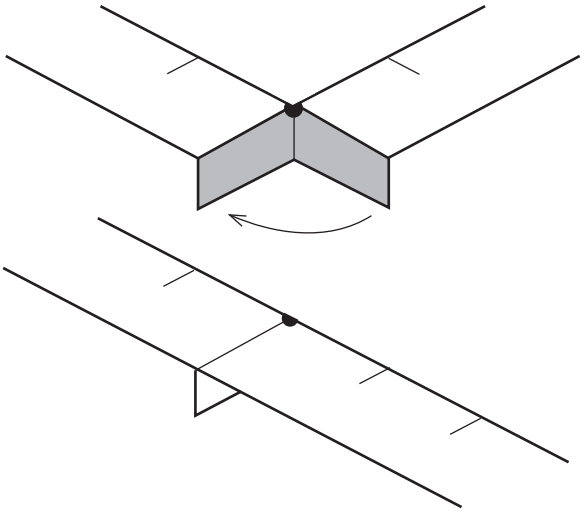


Pattern for Base Five Measuring Tape

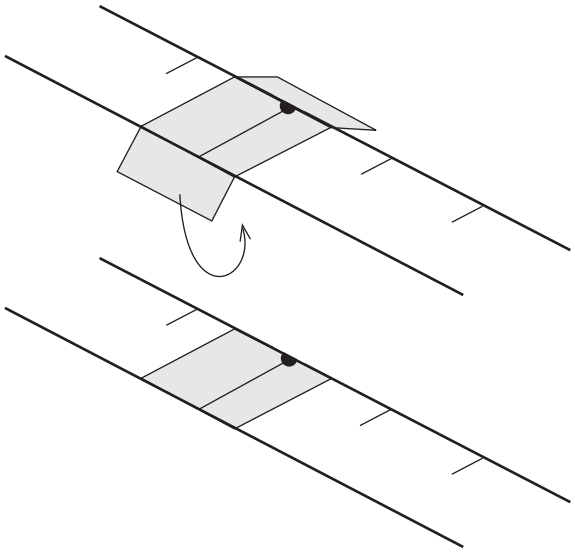
I. Cut along all heavy lines.

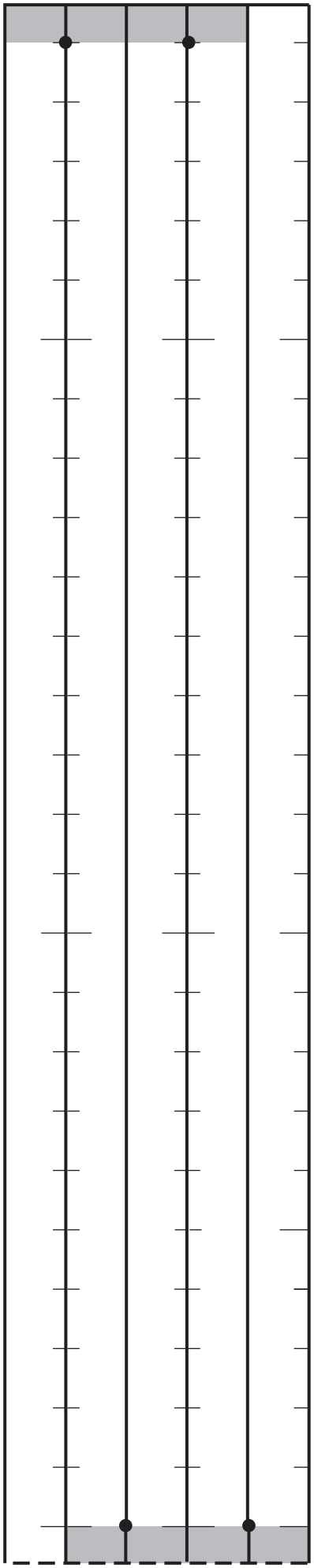


2. Fold in shaded areas:



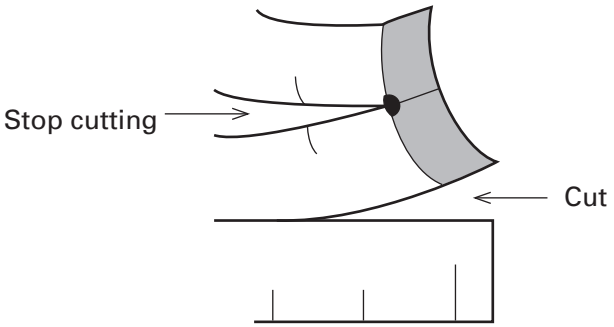
3. Flatten tab and wrap connection with scotch tape:



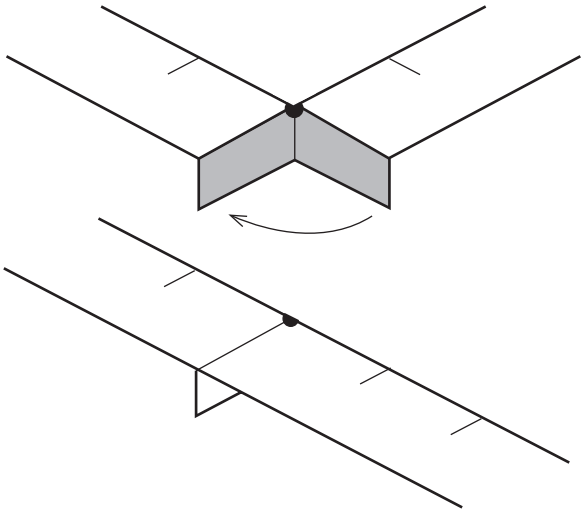


Pattern for Base Ten Measuring Tape

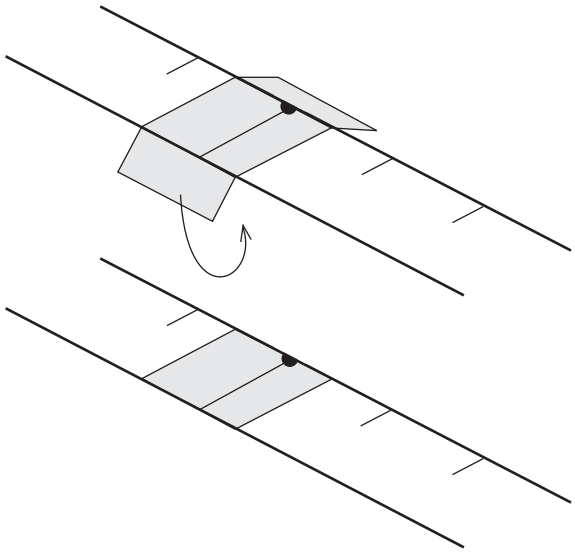
I. Cut along all heavy lines.



2. Fold in shaded areas:

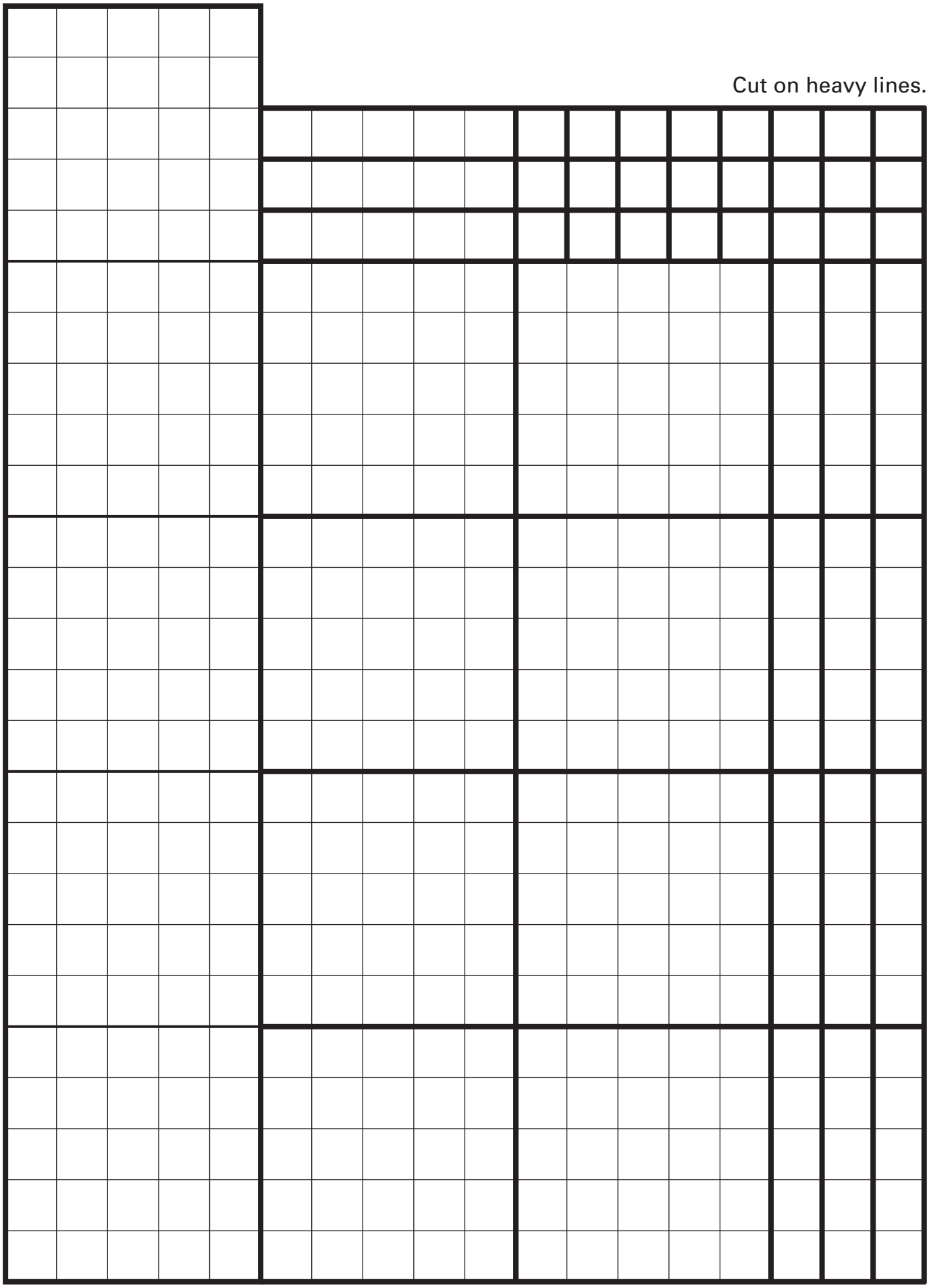


3. Flatten tab and wrap connection with scotch tape:

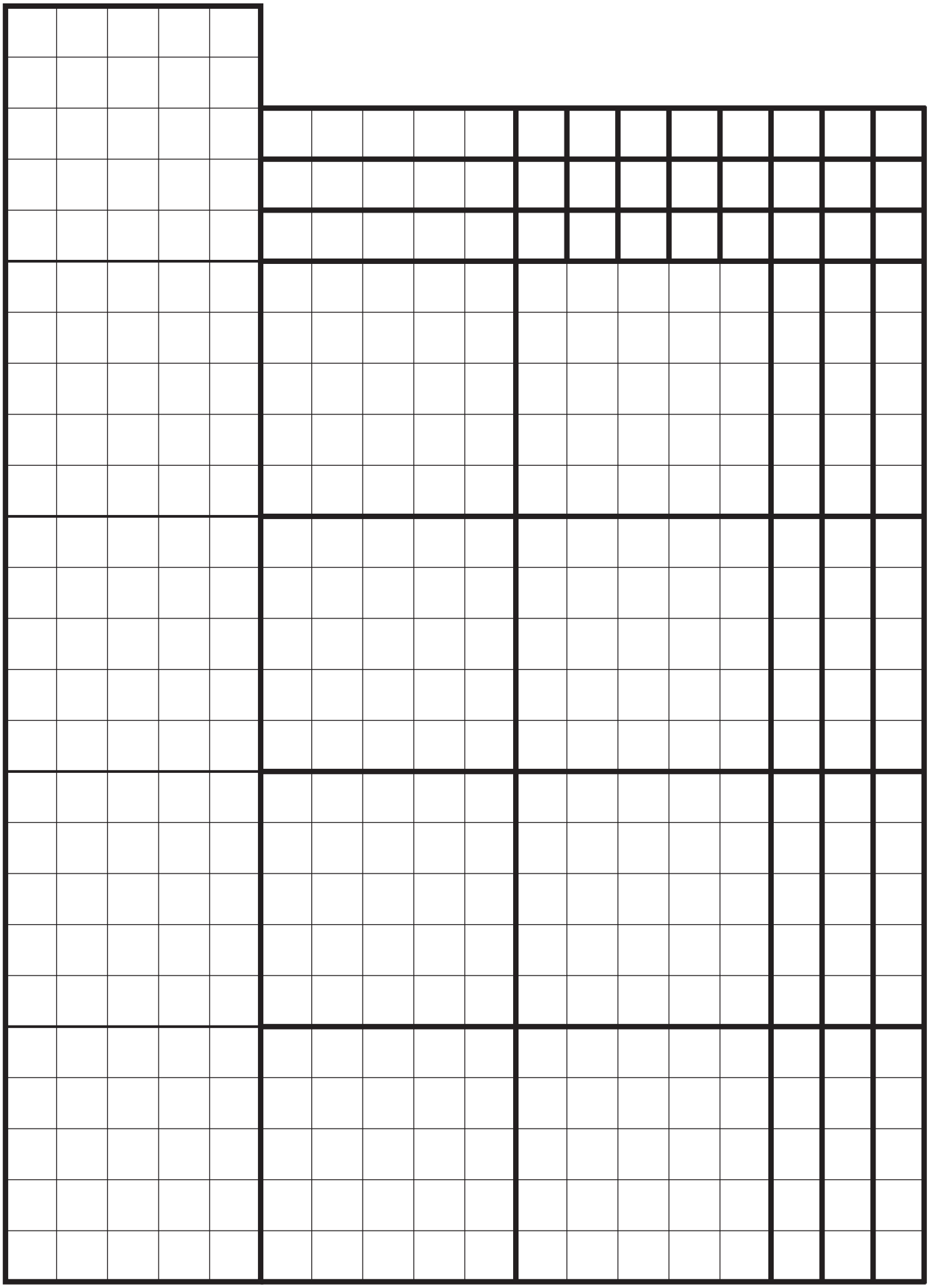


Base Five Area Pieces

Cut on heavy lines.

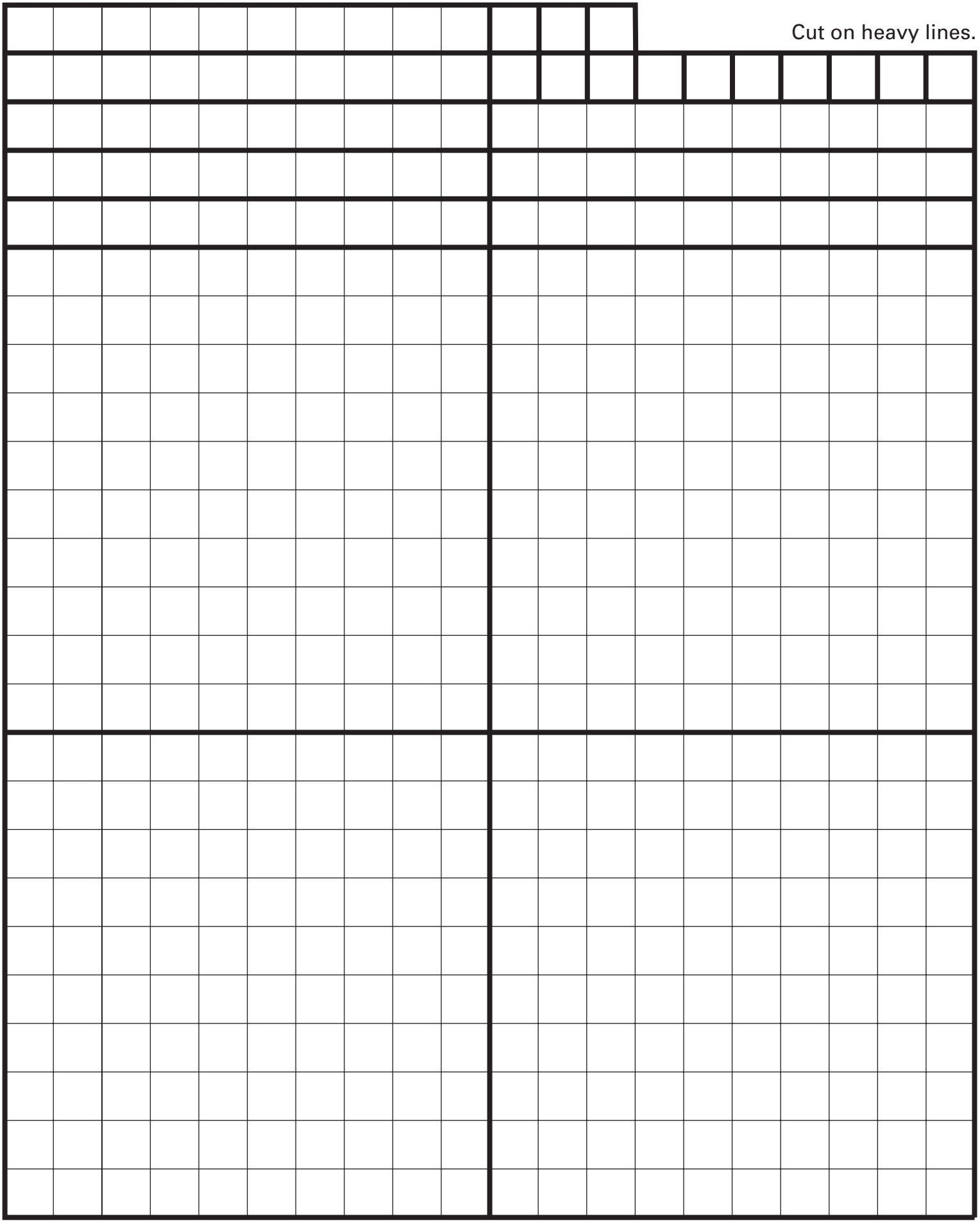


Base Five Area Pieces

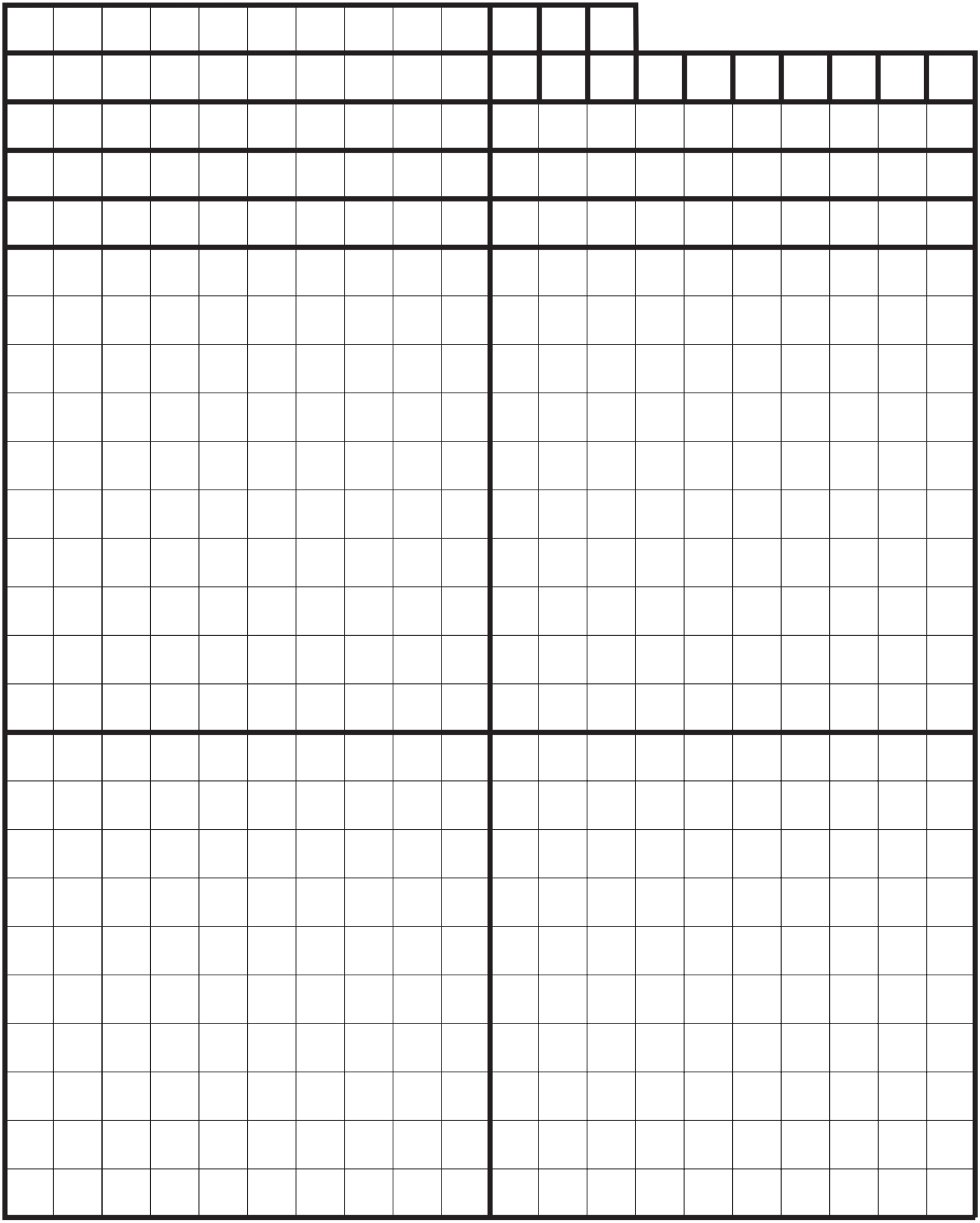


Base Ten Area Pieces

Cut on heavy lines.



Base Ten Area Pieces



Base Ten Area Pieces